BOOK OF ABSTRACTS
LIVRO DE RESUMOS

XX Brazilian Logic Conference

XX EBL

2022

Ciro Russo
Leandro Suguitani
Marcelo D. Passos

Editors

XX BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XX ENCONTRO BRASILEIRO DE LÓGICA

LIVRO DE RESUMOS

XX Brazilian Logic Conference
XX EBL

September 12-16, 2022 – Salvador, Brazil
September 10-11 (**Logic School**)
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Preface

The Brazilian Logic Conference (EBL) is the main event organized by the Brazilian Logic Society (SBL) and it has happened since 1979. The EBL congregates logicians of different areas and the meeting is an important moment for the Brazilian and Latin America research communities to come together and engage in fruitful discussions about ongoing research projects. The areas of Logic covered by EBL spread over Foundations and Philosophy of Science, Mathematics, Computer Science, Informatics, Linguistics, and Artificial Intelligence. The goal of the EBL is to encourage the dissemination and discussion of research papers in Logic in the broadest sense. Previous editions of the EBL have been a great success, attracting researchers from all over Latin America and elsewhere.

In 2022, the 20th edition of the EBL will be held on September 12-16 at the Institute of Mathematics and Statistics (IME) of the Federal University of Bahia (UFBA), in Salvador, the capital city of the State of Bahia. It is the second time that Salvador hosts this meeting, the first was back in 1996. UFBA, one of the most important centers for research in Logic in Brazil, is honored to host EBL's 20th edition. Here logic is a research area in three departments: Mathematics, Computer Science and Philosophy, and the group is constantly growing.

As part of the EBL, the Logic School will be held on September 10-11 and it will feature three tutorials of three and a half hours each. Back in 2008, since EBL's 15th edition, the Logic School is an event that aims mainly at undergraduate and graduate students and connected researchers, but it is also open to anyone who may be interested in Logic. The Logic School offers tutorials on different subject matters to promote academic interaction and stimulate students to be open minded in the fields of Logic.

The call for papers of this 20th EBL expected submissions on general topics of logic, including philosophical and mathematical logic and applications, history and philosophy of logic, non-classical logic and applications, philosophy of formal sciences, foundations of computer science, physics and mathematics, and logic teaching. The abstracts of the talks presented in this volume reflect the plurality of interests in Logic that comes across in the EBL. This volume brings abstracts of 69 session talks, 10 invited speakers conferences, 3 round tables, 7 posters and 3 tutorials of the Logic School.

The 20th EBL has been sponsored by SBL, INCTMat, and CNPq. It also had the support of UFBA, IME, and Instituto Carybé.

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Invited Speakers
Palestrantes Convidados

Applications of combinatorial families

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Abstract

The Schreier family $\mathcal{S} = \{F \in [\mathbb{N}]^{<\omega} : |F| \leq \min F\}$ was introduced by J. Schreier in the 1930's with a motivation coming from Banach space theory. The reason why this family was good for his purposes is related to the combinatorial properties of the family. More specifically, Ramsey theoretic properties of the family $\mathcal{S}$ were usefull to build some peculiar sequence in the Banach space $X_{\mathcal{S}}$, now known as the Schreier space.

More recently, several sorts of combinatorial families (regular families, uniform families, barriers, etc), both in the countable and in the uncountable setting, were used in applications to topology or Banach spaces. In particular, in 2009 W. T. Gowers introduced in his weblog the notion of a combinatorial space. In this talk, we will give an overview of some of the applications in Banach space constructions contained in [1] and [2] and the combinatorial aspects involved therein.

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On the development of Logic in Brazil

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Abstract

In this talk I will present a historical overview of the development of logic in Brazil and will describe the development of contemporary logic in the country with an emphasis on its socio-institutional and interdisciplinary aspects. After a brief introduction recounting the development of logic within the Luso-Brazilian academic milieu, I will mention the work of the first Brazilian authors and groups of scholars who may be considered logicians.

Special emphasis will be given to the emergence of original research in logic in Brazil with the pioneering work of Newton Carneiro Affonso da Costa and the creation of paraconsistent logic. Also highlighted will be the establishment of the Centre for Logic, Epistemology and History of Science (CLE) at the University of Campinas (Unicamp), the creation of the Brazilian Logic Society (SBL), the realization of the Brazilian Logic Conferences (EBLs), and the Brazilian participation in the Latin American Symposia on Mathematical Logic (SLALMs).

We will also recall the more recent initiatives in Brazil related to logic and, in an attempt to capture the various dimensions of the cultivation of logic in Brazil, we will mention recognized groups for teaching and research on logic in the country, their areas of expertise, the periodicals and series of books edited by research centers devoted to logic, and the regular events organized by these centers.

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Charles S. Peirce on Identity: The case of the Existential Graphs

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Abstract

The notion of identity plays a special and almost mysterious role in logic. In this presentation, I examine the ideas on identity as a logical notion developed by Charles S. Peirce in his late diagrammatic logic of the Existential Graphs. Identity was expressed in the Beta Graphs by means of the line of identity. My main claim is that Peirce achieved by the line of identity a rich and useful analysis of existential quantification in the special sense of uniqueness of decomposition, but its function as an icon for identity is problematic. I will show that the line of identity does not correspond to the notion of identity stated later in the standard classical First-Order Logic with identity, but in the context of the Beta Graphs, the line of identity expressed individual identity. I will also show the differences of the formulation of identity in the Beta Graphs with the presentation in the context of the algebra of logic, where identity was regarded as a second-order predicate. Finally, I will argue, more generally, that in the turmoil of ideas at the origins of modern logic, identity was diversely understood, playing different roles.

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A foundation in which numbers count

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Abstract

The notion of number employed in everyday discourse is bound up with the act of counting. Despite this, neither of the two major foundational approaches to arithmetic give a full account of what is involved, conceptually, with the act. The Dedekind-Peano axioms capture the sequential character of the numbers we count with, but do not illuminate what we determine in counting with them. Conversely the how-many questions we answer with counting are given center stage in neo-Fregean approaches, but the central role of number sequences in answering them via counting goes unacknowledged. Taking remarks of Benacerraf in his seminal [1] as a starting point, I present a foundational approach to arithmetic that charts a course between these two poles. Numbers at the most basic level are conceptualized as words that appear in acts of what Benacerraf terms 'intransitive counting.' From this basis a notion of a counting sequence is developed, in terms of which the operations of addition and multiplication are defined, In the resulting theory, the generality of statements such as the commutativity of addition are secured not by mathematical induction, but a version of the pigeonhole principle.

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On Proof Theory in Computational Complexity

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Abstract

The subject *logic in computer science* entails proof theoretic applications. So the question arises whether open problems in computational complexity can be solved by advanced proof theoretic techniques. In particular, consider the complexity classes NP, $coNP$ and $PSPACE$. It is well-known that NP and $coNP$ are both contained in $PSPACE$, but till recently precise characterization of these relationships remained open. Now in joint papers with E. H. Haeusler [1], [2] (see also [3]) we presented proofs of the equalities $NP = coNP = PSPACE$. These results were obtained by appropriate proof theoretic tree-to-dag compressing techniques, as follows.

Recall that by conventional interpretation of ND (: *natural deductions*), derivations are rooted trees whose nodes are labeled with formulas, ordered according to the inference rules allowed; top formulas and the root formula are called assumptions and conclusion, respectively. Proofs are derivations whose all assumptions are discharged [5]. We use more liberal interpretation that allows dag-like derivations interpreted as DAGs (: *directed acyclic graphs*), not necessarily trees. Obviously dag-like derivations can be exponentially smaller than tree-like counterparts (whereas our dag-like proofs require a special notion of correctness). We elaborated a method of twofold horizontal compression of certain "huge" *quasi-polynomial* exponential-weight tree-like proofs ∂ into equivalent "small" polynomial-weight dag-like proofs ∂_0 containing only different formulas at every horizontal level, whose correctness is verifiable in polynomial time by a deterministic TM. First part of compression is defined [1] by plain deterministic recursion on the height that provides us with "small" polynomial-weight dag-like proofs in a modified ND that allows multiple-premise inferences. In the second part [2] we apply nondeterministic recursion to eliminate multiple premises and eventually arrive at "small" dag-like proofs ∂_0 in basic ND, as desired. As an application [3] we consider simple directed graphs G and canonical "huge" tree-like exponential-weight (though polynomial-height) normal deductions (derivations) ∂ whose conclusions are valid iff G have no Hamiltonian cycles. By the horizontal compression we obtain equivalent "small" polynomial-weight dag-like proofs ∂_0 and observe that the correctness of ∂_0 is verifiable in polynomial time by a deterministic TM. Since Hamiltonian Graph Problem is coNP-complete, the existence of such polynomial-weight proofs ∂_0 proves $NP = coNP$ [2], [3]. Now consider problem $NP = ?PSPACE$. It is known that the validity problem in propositional minimal logic is PSPACE-complete. Moreover, minimal tautologies are provable in Hudelmaier's cutfree sequent calculus [4] by polynomial-height tree-like derivations ∂. Standard translation into ND in question yields corresponding "huge" tree-like proofs ∂' that can be horizontally compressed into desired "small" dag-like polynomial-weight proofs ∂_0 whose correctness is deterministically verifiable in polynomial time. This yields $NP = PSPACE$ [2].

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On many-valued modal logics and graded beliefs

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Abstract

Modal logics are well-known formalisms that account for representing and reasoning about notions such as necessity, belief, uncertainty, knowledge, similarity, obligations, time, etc. For instance Halpern's book [9] discusses a familiy of modal logics to reason about uncertainty. On the other hand, many-valued logical systems under the umbrella of the so-called *mathematical fuzzy logic* (in the sense of Hájek [3]) appear as suitable logical frameworks to formalize reasoning with gradual properties, i.e. notions whose satisfaction is a matter of degree. Therefore, if one is interested in reasoning involving both gradualness and some sort of modalities one is led to study systems of many-valued modal logic. In this talk we will mainly survey various families of many-valued or fuzzy modal logic of different nature that have been proposed in the literature to reason about different models of uncertainty in the sense of graded belief [6,8] and about preferences [12], both qualitatively and quantitatively. One of such families consist of two-layer modal systems, where the inner logic deals with the objects (events) over we quantify the uncertainty (that can be classical logic or other modal or many-valued logic), and the outer (many-valued) modal logic is used to reason about the involved modal notion itself (e.g. probabilistic or possibilistic uncertainty). These systems are relatively simple to axiomatise as they allow neither nested modalities nor mixed propositional and modal formulas. Typical and initial examples of these systems are fuzzy probabilistic logics where the outer logic is $[0,1]$-valued Łukasiewicz logic [7,8]. Other families of many-valued modal logics follow a more traditional approach, after Fitting's pioneering work [4], with a classical-like language extended with modal operators and with Kripke-style semantics, where possible worlds and the accessibility relation are many-valued, see e.g. [1,2]. Representative examples of these systems are Caicedo and Rodriguez's minimal modal logics based on Gödel $[0,1]$-valued logic [2], whose KD45-like extension has been recently shown to be adequate to reason about possibilistic uncertainty [10]. Finally, we will discuss different systems in this setting as well as some recent negative results by Vidal about non-axiomatizability of modal logics based on certain many-valued logics [11].

Acknowledgments

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Temporal truth and bivalence: logical remarks on Aristotle's *De Interpretatione 9*

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Abstract

The controversy over the famous Sea Battle Argument, presented by Aristotle in *De Interpretatione 9*, dates back to antiquity. According to the reading labeled as traditional, the conclusion of the argument is the denial of the universal law of bivalence, according to which every statement is true or false.

The label may be inappropriate, as there is historical evidence that this reading was not predominant even among the ancient followers of Aristotle, the so-called Peripatetics. They would have seen it as incompatible with the definition of truth and the sacred law of excluded middle that Aristotle vindicates in Book Gamma of the *Metaphysics*.

This disparaging judgment on the logical theses that the traditional reading finds in *DI 9* is shared by contemporary logicians like Quine, who qualifies them as simply delusional. I believe that this judgment, and the scruples of the Peripatetics, are unfounded.

First, I intend to show that the temporal concept of truth that Aristoteles introduces in *DI 9* is not contrary, but complementary, to the temporally neutral concept of truth he elucidates in the *Metaphysics*. Secondly, I intend to show that the heterodox logical theses sustained in *DI 9* can underpin a systematic and coherent semantics for propositional logic.

More than that, by resorting in an anachronistic vein to concepts and methods peculiar to contemporary logic, I intend to show that a formal semantics can be defined for classical propositional calculus that perfectly reflects the heterodox theses of *DI 9*. It is remarkable that, although this semantics amounts, on the metalinguistic level, to the refusal of the logical universal law of bivalence, it preserves, on the object language level, all instances of the law of excluded middle, as well as everything else that classical propositional logic takes to be a logical truth.

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How to change beliefs about concepts?

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Abstract

With the advancement of technology, a major challenge has become the ability to handle a huge amount of information. Any agent that can be called "intelligent" needs a mechanism to decide what to do with new information received.

Belief revision theory [1, 5] deals with the problem of how to accommodate new information received by an agent. The new piece of information may be inconsistent with what the agent previously believed, so in order to accept it, the agent may have to abandon some of its previous beliefs. The theory can be applied in different areas, such as diagnostics (something is not working as we believe it should be), databases, program specification repair, among others.

In this lecture, I intend to introduce the area of belief revision and show an application in the area of ontology maintenance [3, 4, 6–8]. Ontologies, in Artificial Intelligence, are formal descriptions of concepts and their relationships, facilitating communication between different agents. In particular, we apply belief revision techniques to ontologies described in Description Logics [2].

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Dialectica Categories Revisited

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Abstract

Gödel's Dialectica interpretation was conceived as a tool to obtain the consistency of Peano arithmetic via a proof of consistency of Heyting arithmetic, in the 40s. In recent years, several proof-theoretic transformations, based on Gödel's Dialectica interpretation, have been used systematically (by Kohlenbach and many others) to extract new content from classical proofs, following a suggestion of Kreisel. Thus, the interpretation has found new relevant applications in several areas of mathematics and computer science.

Several authors have explained the Dialectica interpretation in categorical terms. In particular, I have introduced the notion of a Dialectica category as an internal version of Gödel's Dialectica Interpretation in my doctoral thesis, written under Hyland's supervision. This categorical Dialectica construction has been generalized in many meaningful directions, and it has had many applications developed from it, from concurrency theory and Petri nets, from linear logic models of state and games, to Set Theory and 'small cardinals' and 'Kolmogorov-Veloso problems'. Recently the construction has been under scrutiny, as many applications in computing, especially ones using lenses and bidirectional transformations have been discussed, in parallel with applications of the category of polynomials. This is all part of a growing movement of Applied Category Theory, of which the Topos Institute is one of the centers and that we shall also discuss.

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Some topics in the foundations of supervised statistical machine learning

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Abstract

We will discuss some set-theoretical aspects of the algorithms of supervised statistical machine learning. Examples include a problem by Vidyasagar on learnability of classes under non-atomic distributions, as well as a problem of Devroye, Györfi and Lugosi on the existence of a universally consistent learning rule with a universally monotone error.

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Tutorials (Logic School)
Tutoriais (Escola de Lógica)

Interpretations: A logician's toolkit

Canal Ad Infinitum*
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Abstract

First-order theories are objects of undeniable interest to philosophers, logicians and mathematicians. Such theories have three central components: formal language, deductive apparatus and semantics. The formal language is a collection of symbols from which we establish the definitions of term, formula, sentence, free variable, and so on. The deductive apparatus fixes a collection of sentences and establishes the rules according to which new formulas are obtained. From this we define the concepts of axiom, rules of inference, deduction, theorem, etc. First-order structures are a very versatile instrument for fixing the semantics and with then we define, among others, the notion of a sentence be satisfied in a strutucture.

Of course, first-order theories are not just formal games. On the contrary, one of its priorities is to systematically investigate subjects of undeniable interest to logic, mathematics and philosophy.

A logician, when analyzing the deductive apparatus of a first-order theory, must answer questions concerning decidability and consistency. Furthermore, if two different logicians are investigating a given subject, it may be the case that each of them formalizes the subject under study in theories with different languages, axioms and rules of inference. In this case, in what sense can we say that the theories studied by them concern the same subject?

A similar situation occurs when two mathematicians investigate a certain mathematical content, such as arithmetic. One of the mathematicians can study arithmetic from the perspective of the standard structure $(\mathbb{N}, +, \times, 0, 1)$ while the other investigates arithmetic from the structure of finite sets provided with the membership relation $(V_{fin}, \in)$. Since the languages of these structures are distinct, they are not isomorphic and therefore it is reasonable to ask: in what sense do these mathematicians investigate the same theme?

The concepts of interpretation between structures and interpretation between theories allow us to articulate a precise answer to the questions formulated above and will be the subject matters of this mini-course. More precisely, we will present the concept of interpretation between structures and elucidate under which conditions we can say that two non-isomorphic structures are "essentially the same". From the interpretation between structures, we will approach the notion of interpretation between theories and how this notion responds to questions about decidability and consistency of theories. All the concepts presented throughout the mini-course will be accompanied by a lot of examples and applications.

Important: The mini-course will be taught in Portuguese and it is aimed at undergraduate and graduate students who have taken some course in classical logic during their undergraduate studies. The basic references on the subject are [1], [2] and [3] and examples of applications are the pappers [4], [5] and [6].

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Lógicas Multimodais: Completeza, Complexidade e Aplicações

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Resumo

Lógicas Modais são uma família de lógicas não verifuncionais que possuem um operador de qualificação de informação, como 'é necessário que', e vêm recebendo crescente atenção de áreas como a Filosofia, a Ciência da Computação e a Inteligência Artificial dada sua flexibilidade, expressividade e interessantes propriedades computacionais. Lógicas Multimodais são aquelas obtidas combinando variadas lógicas modais em um único arcabouço de raciocínio. Enquanto resultados já clássicos da teoria da correspondência para lógicas modais nos fornecem subsídios para estudar propriedades de lógicas multimodais, estudar a interação entre os operadores de tais lógicas escapam ao escopo de tais resultados. Para tanto, precisamos empregar resultados da área de combinação de lógicas. Nesse minicurso, estudaremos alguns resultados básicos sobre lógicas multimodais, explorando aspectos da teoria da correspondência para lógicas modais, combinação de lógicas e aplicações de lógicas multimodais na Epistemologia Formal e Inteligência Artificial.

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Uma introdução à Teoria dos Modelos de espaços métricos generalizados

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Resumo

A Lógica Contínua foi introduzida nos anos 60 (Chang-Keisler [2]) e redescoberta nos anos 90 (Henson, Iovino, et al [1]) para estudar estruturas baseadas em espaços métricos completos (e.g., Espaços de Hilbert, de Banach), pois o estudo dessas estruturas desde o ponto de vista da Lógica de Primeira Ordem é mal comportada (Shelah-Stern [8]).

Porém, este ponto de vista não desenvolve o estudo de espaços topológicos em geral. Desde o ponto de vista da Lógica de Primeira Ordem, tem uma tentativa de estudo de espaços topológicos (Pillay, Kucera, et al) mas só considera espaços topológicos com algum conteúdo algébrico -e.g., Teoria dos Modelos de Módulos e Grupos Topológicos-.

Os "Quantales" são um tipo de reticulado que generaliza os "Locales" ("espaços topológicos sem pontos") que tem uma compatibilidade com uma estrutura algébrica de monoide (e.g., reticulados multiplicativos de ideais em Anéis e Análise Funcional). Flagg [3] provou que qualquer espaço topológico pode ser entendido como um espaço pseudo-métrico com uma distância valorada num Quantale adequado.

Lieberman, Rosicky e Z. [6] propuseram num contexto geral (análogo às classes não elementares AECs) uma definição de estrutura métrica generalizada baseada em distâncias valoradas em Quantales, generalizando as estruturas na Lógica Contínua.

Depois, Reyes e Z. [7] estudaram esta proposta como uma generalização da Lógica Contínua, obtendo uma caracterização da Lógica Contínua sob algumas condições adicionais para os Quantales.

Na primeira sessão deste mini curso, falaremos sobre alguns temas básicos da Lógica Contínua e dos Quantales.

Na segunda sessão, falaremos sobre a nossa proposta de Lógicas Quantale-valoradas e a nossa caracterização da Lógica Contínua neste ponto de vista, utilizando um resultado de Iovino [5] que caracteriza a Lógica Contínua como a lógica maximal no contexto métrico que satisfaz o teorema das cadeias enumeráveis de Tarski-Vaught e o teorema de compacidade. Tirando alguma destas condições adicionais para os Quantales para obter estes resultados, as quais são satisfeitas pelo intervalo unidade [0,1], obtemos novas lógicas.

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Round Tables
Mesas Redondas

Mulheres e lógica: ideias, projetos e ações!

Christina Brech (USP) Itala D'Ottaviano (UNICAMP)
Manuela Souza (UFBA) Sara Uckelman (Durhan University) Renata Wassermann (USP)

Resumo

Em 2019, organizamos, como parte das atividades do XIX EBL em João Pessoa, uma mesa redonda com o título: Mulheres na Lógica (e no Brasil). A mesa redonda foi coordenada por Gisele Secco, Cláudia Nalon, Valeria de Paiva e Elaine Pimentel, e contou com uma presença expressiva de participantes do congresso.

Essa foi uma ideia, que acabou se refletindo em um projeto: dali nasceu o Lógicas Brasileiras (https://logicasbrasileiras.wordpress.com/), coletivo que visa a divulgação de projetos de pesquisa e ensino, história e memória de, com, para e relacionados a mulheres brasileiras que fazem ou fizeram lógica.

A partir desse projeto, desenvolvemos uma série de ações, como a participação no #ShutDownLogic, realização de entrevistas com lógicas brasileiras, e a organização do Lógica e Representatividade e do Dia Carolina Blasio.

No EBL de 2022, propomos continuar com a discussão de ideias, projetos e ações passadas e que estão por vir, sobre/com mulheres que fazem/estudam/se interessam por lógica no Brasil e no mundo. As convidadas são:

- Christina Brech (USP)
- Itala D'Ottaviano (UNICAMP)
- Manuela Souza (UFBA)
- Sara Uckelman (Durhan University)
- Renata Wassermann (USP)

Lógicas, ensino e extensão

Andreas B.M. Brunner (UFBA) Camila Jourdan (UERJ)
Sara Uckelman (Durhan University) Marcos Silva (UFPE) Petrucio Viana (UFF)
Bruno Lopes (UFF) Gisele Secco*(UFSM)

Resumo

Por sua natureza formal e sua hereditária pretensão de universalidade, a lógica é um campo de conhecimento ou disciplina que se engendra e se transmite de diferentes modos, compartilhados por grupos de pessoas em determinados contextos históricos. Ademais, essas variações nos modos de engendramento e transmissão são indexadas pelas áreas nas quais a lógica recebe aplicação: computação, filosofia, matemática e linguística (e suas diversas variações). Por fim, a história contemporânea da disciplina atesta uma crescente ampliação da pluralidade de perspectivas desde as quais se criam e se articulam os mais variados sistemas lógicos. Ocorre também em lógica, vale notar, um movimento de maior atenção para tradições de pensamento não helenocêntricas, culturas ditas "outras".

A despeito de tamanha diversidade intrínseca, e tal como as STEM e a filosofia, boa parte das comunidades de prática em lógica estão superpovoadas de pessoas de só um gênero, ou uma só etnia, ou uma só cor, e algumas poucas classes. (Algumas diriam mesmo que ser coisa de homem é um traço característico da lógica ela mesma: [1]). Os efeitos dessa supremacia são invariavelmente perniciosos, de modo que a falta de diversidade de gênero, raça ou etnia e classe entre grupos de gente que faz lógica (lógicas e lógicos, professores de lógica, estudantes de lógica, amantes de lógicas etc..) não passa mais despercebida – como é atestado pelas recentes conversações abertas sobre o tema [2], [3] e [4].

A circulação de materiais didáticos e sugestões pedagógicas e curriculares decorrente dessa maior atenção a questões de interseccionalidade e ensino de lógica é admirável (veja-se [5], [6] e [7] para alguns exemplos), mas ainda há muito o que fazer para que as paradas e viradas culturais cuja consolidação testemunhamos por todo lado comecem a se radicar também na lógica: tanto nos modos como a ensinamos quanto nas narrativas que transmitimos sobre a história de nossa disciplina.

Outra maneira de ampliar os horizontes da prática lógica em voga é a divulgação de materiais audiovisuais em plataformas de comunicação em massa. Florescem cursos, minicursos, aulas, vídeos informativos de toda sorte (como palestras e explicações de resultados lógicos strictu sensu, aulas de introdução, história e fenomenologia da lógica, avaliação de discursos cotidianos etc.) [8], [9], [10] e [11]. Neste cenário, parece evidente que o campo da lógica (como disciplina, como prática, como ofício), tem a ocasião de aperfeiçoar-se significativamente.

Essa mesa redonda pretende dar continuidade às discussões realizadas no último EBL, abordando, a partir das experiências docentes de seus participantes, algumas questões:

- Quais as diferenças mais relevantes entre ensinar lógica em computação, filosofia, matemática? O que há em comum?

- Qual a importância da inclusão de elementos de história e filosofia da lógica nas situações didáticas das diferentes áreas?

- Desde quais perspectivas a história da disciplina pode ou não ser ignorada sem prejuízo para a pretensão de mais diversidade humana na prática lógica? Quais são, em cada caso, as razões para tal?

- O que sabemos sobre o ensino de lógica nas áreas que não as de nossa especialidade? Há estudos sistemáticos sobre o tema nessas áreas?

- Quais experiências (de ensino e extensão) podem servir de amostra ou fomento para a continuidade das conversas sobre ensino, divulgação e diversificação do campo da lógica?

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Wittgenstein on Gödel's incompleteness theorems

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Abstract

Despite the initial negative reception of Wittgenstein's remarks on Gödel, some recent works have appeared with the purpose of contextualizing and offering a more positive reading of these remarks. In this context, this round table proposes to discuss, from different points of view, Wittgenstein's remarks on Gödel afresh. Our main goal is to bring to light some pivotal aspects of Wittgenstein's philosophy of mathematics and discuss i) how they are necessary for an accurate understanding of the published passages on Gödel; ii) to what extent they are reasonable to a standard logician.

Wittgenstein's remarks on Gödel in the light of his anti-revisionistic constructivism

Anderson Nakano (PUC-SP)

The purpose of my talk will be to revisit Wittgenstein's remarks on Gödel (in particular, RFM, I App. III, and RFM, VII §§19-22) with the intention to test a certain interpretation of Wittgenstein's philosophy of mathematics. This interpretation focuses on articulating two apparently opposing tendencies in his thought. The first is his constructivism, which conceives of the mathematical activity as an activity of construction of mathematical "objects" (numbers, figures, proofs, etc.). The other tendency is his confessed anti-revisionism. These two tendencies are seemly opposite because usually constructivism is developed in such a way as to lead to a revision of mathematical practice (e.g., denial of the law of excluded middle, as well as exclusion of non-predicative definitions, etc.). My interpretation, however, provides a way to reconcile these two tendencies. I maintain that, if the proposed interpretation is correct in its main lines, it may shed some light on Wittgenstein's thinking about Gödel's result and, consequently, be helpful for a better exegesis of Wittgenstein's text.

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Returning to Wittgenstein on Gödel: between prose and proof

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Much is already learned from Wittgenstein's considerations of mathematics if we understand that he is always suspicious when a concept is partially (and only partially) modified to avoid a paradoxical conclusion. Such distrust intends to separate prose from proof, that is, what concerns a demonstration from the philosophical interpretation of calculus for Wittgenstein. Particularly about the ill-regarded Wittgenstein's comments on Gödel's theorem, he considers that the realist prose about the proof already supposes that the demonstrability of a formula can be relative to the system, although the truth cannot. In this presentation, following Wittgenstein's comments on Gödel, we will discuss what it means to say that the relationship between demonstrability and truth cannot be fully maintained, although it needs to be partially maintained. Through this approach, we believe we can better understand how Wittgenstein problematizes the requirement that metamathematical statements mirror arithmetic statements. To return on these analysis, the readings of Juliet Floyd and Hilary Putnam will also be taken into account, as well as the discussions on the subject made by Charles Sayward and Stuart Shanker.

Theorems and Philosophical Problems: the Wittgenstein-Gödel case

Luiz Carlos Pereira (UERJ/PUC-Rio)

Johann von Neumann begins his contribution to the Königsberg debate with an impressive diagnosis of the foundational situation in the very beginning of the thirties: "Critical studies of the foundations of mathematics during the past few decades, in particular Brower's system of "intuitionism", have re-opened the question of the origins of the generelly supposed absolute validity of classical mathematics. Noteworthy is the fact that this question, in and of itself philosophico-epistemological, is turning into a logical-mathematical one." The optimistic mood in Formalist circles at the time was based on the belief that a logical-mathematical result would lead to the solution of a philosophical problem. This mood changes dramatically with Gödel's incompleteness theorems: if a philosophical program could be reduced to (or depended on) the formulation of a mathematical result, a proof of the impossibility of this result would fatally lead to the failure of the philosophical program itself. But in what sense can we say, that a philosophico-epistemological question can be turned into a logical-mathematical one? If the mathematical result is not the (or a part of the) answer to a philosophico-epistemological question, then it is just a mathematical result (we should not forget the lesson: if a philosophical problem can be solved by a theorem, then what we have shown is just that it was not a philosophical problem after all). In this short presentation I would like to explore this relation between "theorems" and philosophical questions to discuss some of Wittgenstein's remarks on Gödel's proof and consistency.

Talk Sessions
Comunicações

Autorreferência e Circularidade

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Resumo

Certos paradoxos lógicos, como o do Mentiroso, de Russell, de Grelling-Nelson e outros, foram categorizados sob o rótulo "paradoxos de autorreferência". Em vista disso, parece perfeitamente natural perguntar "o que é autorreferência? Por que tais paradoxos se encaixam sob tal rótulo?"Entretanto, nota-se que os lógicos dedicaram pouca atenção ao problema: muito foi dito, é claro, a respeito de ferramentas lógicas e matemáticas capazes de produzir sentenças autorreferentes[1], mas não muito sobre como definir o conceito formalmente. Algo similar ocorre com o conceito de circularidade: embora amplamente utilizado na lógica, este é raramente definido, o que torna as discussões que o envolvem por vezes obscuras e inconclusivas. Meu objetivo nesta apresentação é, portanto, fornecer definições formais dos conceitos de autorreferência e circularidade e reavaliar, a partir delas, o status de certos paradoxos; em particular, defenderei que o Paradoxo de Russell não deve ser considerado autorreferente, mas sim circular[2], e que o Paradoxo de Yablo não é nem autorreferente nem circular, preervando então a intenção original de Yablo[3].

Antes de partir para minha proposta, exibirei duas definições de circularidade e autorreferência compiladas por Leitgeb [2] que ilustram os modos como os lógicos geralmente entendem estes conceitos. A primeira é a da circularidade como ponto fixo – talvez a mais comumente encontrada em textos de lógica[4]. Mostrarei que esta definição deve ser rejeitada, por ser, em última instância, trivial. A segunda é a que chamo de 'autorreferência incompleta' – o nome se dá pois, apesar de estar no caminho correto para uma definição satisfatória, ela não abrange a autorreferência em todo tipo de sentença. Em certo sentido, minha proposta procura expandir a definição incompleta.

A idéia da definição incompleta é de que uma sentença deve se referir ao que seus termos singulares se referem, e nada mais. Pode-se assim definir:

$$\mathit{ref}_1(x, y) \leftrightarrow_{df} x \textit{ é uma sentença} \wedge \exists z(z \textit{ é um termo singular} \wedge x \textit{ contém } z \wedge z \textit{ ref } y)$$

E a autorreferência é definida como esperado:

$$\mathit{selfref}_1(x) \leftrightarrow_{df} x \ \mathit{ref}_1 \ x$$

Um dos problemas da definição sugerida por Leitgeb é que a relação ref é mantida indefinida. Durante a fala, irei definir também o que entendo por referência: tomo-a como uma relação *semântica*, que se dá entre palavras e os objetos que estas denotam. Esta é uma ideia bastante natural – afinal, para conhecer os referentes de uma frase, é preciso entender o *significado* de seus termos. Portanto, minha concepção de referência se apoia no conceito modelo-teorético de *interpretação* (I). Dada uma interpretação I de uma linguagem $\mathfrak{L}$, tal que $\mathfrak{L}$ está contida no domínio de I, o termo t refere-se ao objeto u se e somente se $I(t) = u$. A partir desta definição, formulo o conceito de autorreferência: dizemos que uma sentença é autorreferente se a interpretação atribuída a um de seus termos é a própria sentença; isto é, S é autorreferente se e somente se, dados um termo t que ocorre em S e interpretação I, $I(t) = S$.

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[1]Ver, por exemplo, [4]
[2]Posição defendida também por Roy Cook, [1], nota de rodapé 32.
[3]Vide [5].
[4]Por exemplo, em [3].

A definição captura o mecanismo ocorrido no Paradoxo do Mentiroso, tal como descrito abaixo (sendo T o predicado de verdade e $\neg$ o símbolo de negação):

(L) $\neg T(\ulcorner L \urcorner)$

No caso, a autorreferência é codificada pelo termo L na sentença; ou seja, $I(L) = L$.

Keywords. Autorreferência, circularidade, paradoxos.

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Interpretação co-homológica e homotópica da teoria de modelos

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Resumo

Visamos estabelecer uma leitura co-homológica e homotópica da noção de modelo nas lógicas partindo das reflexões formais de Guitart. Em [6], [7] e [8], Guitart esboça uma programa de geometrização da lógica. Esse projeto passa por diversas considerações que levam Guitart a defender o slogan:

$$\text{Lógica} = \text{Álgebra homológica.}$$

De fato, Guitart mostra em [8], via teoria dos esboços inaugurada por Ehresmann em [5] e os métodos simpliciais desenvolvidos por André em [1], que dados uma linguagem de primeira ordem $\mathcal{L}$, uma fórmula φ nessa linguagem, e uma $\mathcal{L}$-estrutura M, é possível atribuir grupos de homologia e cohomologia

$$H_n^{\varphi}(M), \qquad H_{\varphi}^n(M)$$

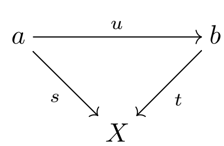
tais que, se $M \models \varphi$, i.e., M é um modelo da fórmula φ, então os grupos $H_n^{\varphi}(M)$ (resp. $H_{\varphi}^n(M)$) são isomorfos aos grupos de homologia (resp. cohomologia) do ponto:

$$H_0^{\varphi}(M) = \mathbb{Z}, \quad \text{e} \quad H_n^{\varphi}(M) = 0 \quad \text{para} \quad n \neq 0$$
$$H_{\varphi}^0(M) = \mathbb{Z}, \quad \text{e} \quad H_{\varphi}^n(M) = 0 \quad \text{para} \quad n \neq 0.$$

É notável que os grupos de homologia e co-homologia acima medem a obstrução para M ser um modelo de φ: eles medem o desvio da relação $M \models \varphi$. A técnica utilizada por Guitart passa pelo seu método dos diagramas localmente livres e pela construção do esboço associado à uma linguagem de primeira ordem. Contudo, o argumento geral independe dessa apresentação, sendo estritamente categorial: dados uma categoria (pequena) A, que é uma subcategoria *plena* de uma categoria (localmente pequena) $\mathcal{C}$, e um funtor $T : A \longrightarrow \mathcal{A}b$ de A a valores de grupos abelianos (por exemplo, obtido da composição de um funtor de esquecimento, $U : A \to Set$, com o funtor de grupo abeliano livre, $L : Set \to \mathcal{A}b$), é possível atribuir para cada objeto X de $\mathcal{C}$ a homologia e a cohomologia da categoria pequena A/X com coeficientes em T:

$$\mathsf{H}_*(A/X; T), \qquad \mathsf{H}^*(A/X; T).$$

Lembramos que a categoria A/X é formada pelos pares (a, s) tais que a é um objeto de A e $s : a \to X$ é uma flecha de a para X em $\mathcal{C}$, e os morfismos de A/X são definidos de modo evidente por triângulos comutativos:



Os grupos de homologia $H_n(A/X; T)$ e cohomologia $H^n(A/X; T)$ são tais que, se X é um objeto de A, então

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$$H_0(A/X;T) = TX, \quad \text{e} \quad H_n(A/X;T) = 0 \quad \text{para} \quad n \neq 0$$
$$H^0(A/X;T) = TX, \quad \text{e} \quad H^n(A/X;T) = 0 \quad \text{para} \quad n \neq 0.$$

Além disso, a homologia $\mathsf{H}_*(A/X;T)$ e a cohomologia $\mathsf{H}^*(A/X;T)$ dependem apenas do *tipo de homotopia* de A/X. Logo, se considerarmos o funtor constante $k_{\mathbb{Z}} : A \to \mathcal{A}b$ fixado em $\mathbb{Z}$, a categoria $\mathsf{Mod}(\mathcal{L})$ das $\mathcal{L}$-estruturas, e a subcategoria plena $\mathsf{Mod}[\varphi]$ formada pelos objetos que são modelos da fórmula φ, podemos aplicar o método anterior e concluir o resultado de Guitart.

A generalidade da homologia e cohomologia das categorias indica que a interpretação co-homológica visada por Guitart não depende (de forma essencial) da particular semântica em lógica de primeira-ordem e mesmo da lógica de primeira-ordem, pois pode ser formalizada em qualquer contexto semântico para qualquer noção de lógica (lógica modal, lógica proposicional, lógicas de ordem superior, lógica em topos, etc.).

A teoria das instituições desenvolvida por Diaconescu em [4] é uma teoria de modelos abstrata, onde a noção de satisfabilidade $\models$ é primitiva. A abrangência dessa teoria é bastante elevada, pois a teoria das instituições trata não apenas da lógica de primeira-ordem e da lógica proposicional, mas de todo o zoológico de lógicas que aparecem nas ciências da computação. Além disso, a teoria das instituições é formalizada já em linguagem categorial. Portanto pretendemos estender os argumentos de Guitart para o quadro das instituições, e mostrar que seu slogan, que estabelece uma igualdade entre lógica e co-homologia, é muito mais profundo do que parece à primeira vista, e admite uma formalização matemática precisa na teoria da homotopia categorial, tão impulsionada por Grothendieck (ver [10], [3], [2]).

Por fim, buscamos propor um conceito co-homológico (mas também homotópico) de equivalência elementar, que generaliza a definição tradicional, e que potencialmente tem consequências relevantes em lógica (ver também [9]). Expandindo nesse ponto, visamos a introdução das técnicas de (co)homologia como uma forma de remediar a natureza rígida da relação de satisfação. Com efeito, a satisfação, à la Tarski, é uma relação binária, não admitindo uma noção de "satisfatibilidade" intermediária. Os métodos (co)homológicos poderiam preencher essa lacuna, criando uma relação mais fina de equivalência elementar. Na apresentação, ilustraremos este ponto por meio de exemplos matemáticos.

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Adding Non-monotonic Reasoning to an Intuitionistic Description Logic

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Abstract

We report work in progress on the design of a logical language intended for representation and reasoning about normative ontologies. We are currently investigating the fundamentals for adding non-monotonicity to an intuitionistic description logic, iALC [1–3, 8]. Our main concern at the moment is how we can add non-monotonicity to it in the simplest and most accurate possible way, especially considering the domain of law, which is the intended usage of this logic.

The logic iALC is an intuitionistic description logic that was tailor-made to model and reason over the domain of law, according to the principles of Kelsenian jurisprudence, which is best represented by the intuitionistic aspect of the logic as opposed to a classical description logic. It has a Sequent Calculus (SC) and a Natural Deduction (ND) systems formalised in order to realise monotonic reasoning. This aids in the legislative process of finding possible antinomies between existing laws and the new ones to be created. However, not all of the processes involving law can be represented monotonically, such as the judicial process.

In a court of law, different parties present law-based arguments - sustained by different pieces of evidence and statements from parties involved in the cases in question - against one another in front of a judge or a jury as to convince them of their thesis. During this process, both parties present new evidence and arguments that aim to change the conclusions to which the judge would arrive. Faithful modelling of this process in its entirety is not naturally monotonic: Since it is based on the addition of new information, the conclusion may change.

The current formalisation of iALC deals only with monotonic reasoning, but it would be interesting for it to be able to deal with aspects of law other than modelling and legislation, such as the judicial process stated above, and the juridical process, for instance. The tools we have for it as of today can be expanded to include non-monotonic reasoning, and we intend to increment both the language (with extra operators) and the ND system (with non-monotonic rules) at first. This ensures that the monotonic fragment of iALC remains intact as we elaborate on its non-monotonicity.

There are some formal techniques for dealing with non-monotonicity such as Reiter's default [14], abductive inference [9], and defeasible reasoning [11, 12], among others. By following Reiter's approach, one can arrive at infinite extensions[1] - which only makes proof search harder. Abductive inference requires a very rich and thorough metatheory in order to define what would be the *best* conclusion to derive from each situation. And, finally, with defeasible reasoning there is the question of how to add a rule of specificity[2] (or even *whether* to add it [13]).

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[1]Let a Default Theory consist of W, a set of formulas and D, a set of default rules. Then, an extension E of (D, W) is the smallest set of formulas containing W, closed under classical consequence and the rules in D that are applicable.

[2]This rule can be thought of as subclass inheritance preempting class inheritance i.e. if a rule is more *specific*, it has precedence.

We, however, believe that it is better for us to have an approach similar to [4, 6, 7], in which the authors add a typicality operator to the classical description logic $\mathcal{ALC}$. Many of the previous mentioned problems seem to be easier solved in our context by adding a similar operator to the language of iALC.

In order to better suit these new non-monotonic operator and rules to the domain of law, we have also been investigating which kind of non-monotonicity to approach. Works such as [10], in which the author explores the ways in which non-monotonicity can be used in legal reasoning, are of great interest to our work. Some of the decisions we have to make are, for example, whether to choose sceptical *versus* credulous reasoning [5]. In this case, we are leaning towards sceptical for it seems to be more adequate by aiming to preserve soundness, which avoids potentially-unwanted conflicting conclusions. Another example would be how to deal with so-called *zombie arguments* [15], either via ambiguity-propagating or ambiguity-blocking reasoning.

The main contribution of this article is to point out the most appropriate way of extending iALC to a non-monotonic logic regarding legal process representation and parsimonious changing.

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Expanding the Leibniz Hierarchy

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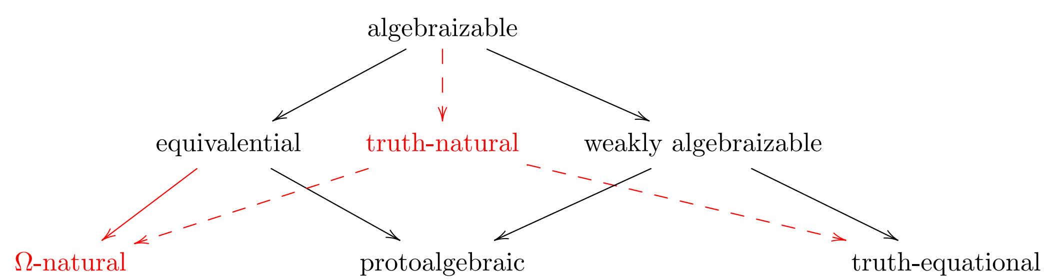
Abstract

In this work, we consider (propositional) logic as a pair consisting of a formula algebra Fm defined in a signature Σ (a set of disjoint connectives and constant symbols), together with a structural Tarskian consequence relation $\vdash$. There is also an added condition of finitarity, but in the most abstract sense of logics, this is not a necessary condition for the definition; although logics that do have it, called finitary logics, have traditionally been given more attention than non-finitary logics, for historical reasons. In addition, our work is based mostly on the theory of logical matrices, while also sharing some ideas with the more recent category-theoretical approach involving the notion of filter-pairs, in line with [2], at the end.

The Leibniz Hierarchy is a classification system for propositional logics expressed in terms of their behavior regarding the Leibniz operator, one of the most important concepts in the field of Abstract Algebraic Logic. According to [3], the main classes of this Hierarchy are *protoalgebraic*, *truth-equational*, *equivalential*, *weakly-algebraizable* and *algebraizable*, each having its own definitions and characterizations with respect to the Leibniz operator.

We propose that there is a new class of logics that can be placed in the Hierarchy, one that should sit directly below equivalential logics, and beside protoalgebraic logics. Since one of the characterizations for equivalential logics involving the Leibniz operator is simply being a protoalgebraic logic together with the property that the Leibniz operator commutes with inverse substitutions/homomorphisms, the initial question that sparked this work was: is there a well-defined class of logics hidden in the Hierarchy that "intersects" with protoalgebraic to form equivalential? If we look at the other side of the Hierarchy, we can see that this is very much the case where protoalgebraic logics meet truth-equational logics, resulting in weakly-algebraizable logics. This new class that we are interested in is exactly the collection of all logics whose Leibniz operators have this commutative property, which can be seen on the bottom left of the diagram (red indicates new additions to the Hierarchy):



To this end, we show that this is a proper class, and notably distinct from the previous ones, by presenting the example of $S^{\Box}$, a finitary logic of type $\Sigma = \{\Box\}$, where $\Box$ is a unary connective, defined only by the rule of inference "$\varphi \vdash \Box\varphi$" and no axioms. This logic is neither protoalgebraic nor truth-equational (hence it is none of the others as well) because it does not have theorems and it is not the inconsistent or almost-inconsistent logic [1], but its Leibniz operator is shown to indeed commute with inverse substitutions. The reason for this is precisely

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the “simple” nature of its formulas, which is to say that every formulas has no more than a single variable, and substitutions/homomorphisms seem to behave well with this kind of configuration.

Next, in a very natural way we generalize this example by showing that all logics defined in a signature that contains only connectives of arity at most 1 necessarily fall into this new class that we are discussing. It is a very intuitive step, taking into account that the reason argued above still applies to any logic defined in these terms.

Moreover, we can identify other connections between this new class and the ones already known, such as pointing at the class given by truth-equational logics whose Leibniz operator commute with inverse substitutions that are not necessarily protoalgebraic, i.e. the “infimum” of truth-equational and the new class, so to speak (at the center of the diagram above), which in turn expands the Hierarchy even further. Here we show another example, albeit slightly more artificial than the previous one, because we resort to the construction of a logic exactly like the previous example, but now with an added axiom “$\vdash \Box\Box x$”, so as to induce truth-equationality into a logic already belonging to this new class, by way of the defining set of equations $\varepsilon(x) = \{\langle x, \Box\Box x\rangle\}$.

In conclusion, we also discuss briefly the implications and possible interpretations of the Leibniz Hierarchy and the apparent three components that make up the class of algebraizable logics. As it stands, the protoalgebraic branch appears to be related to the syntactic component, the same way that the truth-equational branch is related to the semantic component. Now, with the inclusion of this new class and its branch’s quick exploration, we seem to shed a bit more light on what may be referred to as the categorial component of algebraizable logics, in that the commutativity of the Leibniz operator turns it into a natural transformation. Explicitly, Ω defines a natural transformation from the functors Fi_l to $Co_{\mathbf{K}}$, where for every $F \subseteq B$ and $h \in hom(A, B)$, we have $\Omega^A(h^{-1}[F]) = (h \times h)^{-1}[\Omega^B(F)]$, that is, the following diagram commutes:

$$\begin{array}{ccc} Fi_l(A) & \xrightarrow{\Omega^A} & Co_{\mathbf{K}}(A) \\ {\scriptstyle h^{-1}}\uparrow & \circ & \uparrow{\scriptstyle (h\times h)^{-1}} \\ Fi_l(B) & \xrightarrow[\Omega^B]{} & Co_{\mathbf{K}}(B) \end{array}$$

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A tese de normalização sobre identidade de provas sobre o pano de fundo da tese de Church-Turing

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Resumo

Esta comunicação consistirá em uma breve apresentação e discussão do artigo [4]. O texto consiste em uma reflexão metodológica sobre abordagens formais da questão da identidade de provas de um ponto de vista filosófico. Primeiramente, é oferecida uma caracterização da questão da identidade das provas, seguida de uma breve reconstrução da chamada tese da normalização, proposta por Dag Prawitz em [1], na qual são apresentados alguns de seus traços matemáticos e conceituais centrais. Em seguida, uma comparação entre a tese da normalização e a mais conhecida tese de Church-Turing sobre computabilidade é realizada em três partes principais: a primeira dedicada a destacar algumas das analogias entre elas; a segunda, suas diferenças mais notáveis; e a terceira, as possíveis relações de dependência entre elas. Com base nessas considerações, algumas observações finais sobre o potencial da tese da normalização e semelhantes abordagens à questão da identidade de provas são feitas na última seção.

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A Family of Monoidal Structures on the Category of $\mathscr{Q}$-Sets for Commutative and Integral Quantales

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Abstract

In the 1970s, the topos of sheaves over a locale/complete Heyting algebra $\mathbb{H}$ was described, alternatively, as a category of $\mathbb{H}$-sets ([1], [2]). Right-sided idempotent quantales have long been studied as candidates for substituting Locales/complete Heyting Algebras in the standard construction of $\mathbb{H}$-Sets ([3], [4]). In this work we consider an "orthogonal" [1] class quantales – that of the integral/semicartesian quantales (which are also commutative). While the choice of idempotency and right-sidedness is justified by a large class of examples of such quantales of interest – namely, the set of closed right-ideals of a given C^* algebra – our choice of integrality and commutativity could be seen as justified as embodying both the quantales of the ideals of commutative unital rings and $([0,1], \leq, \cdot)$ – which is isomorphic to $([0,\infty], \geq, +)$, thus we generalize metric spaces in a sense.

The definition of $\mathscr{Q}$-set for one such quantale is a direct generalization of the one for complete Heyting algebras, namely it is an underlying set X endowed with a function $\delta : X \times X \to \mathscr{Q}$ satisfying

i) $\delta(x,y) = \delta(y,x)$

ii) $\delta(x,y) \otimes \delta(y,z) \leq \delta(x,z)$

iii) $\delta(x,x) \otimes \delta(x,y) = \delta(x,y)$

which directly correspond to pseudo-metric space axioms, with the modification that $\delta(x,x)$ need not be $\perp$. The literature (and we as well) commonly denote $\delta(x,x)$ by $\mathrm{E}\,x$ – to be read as "the *extent* of x": this is always a member of $\mathrm{E}\,\mathscr{Q}$, the (locale) of all idempotents of the quantale $\mathscr{Q}$.

We consider morphisms between $\mathscr{Q}$-sets, $f : (X,\delta) \to (X',\delta')$, to be ordinary set-theoretical functions between the underlying sets that: i) preserve extents; ii) increase (possibly not strictly) δs. *i.e.* $\mathrm{E} = \mathrm{E}' \circ f$ and $\delta \leq \delta' \circ (f \times f)$. The resulting category has rather good properties – being cocomplete; complete; enjoying a regular monos classifier, *etc.*

In our research we have been met with multiple ways of defining a monoidal structure on the category with the same "spirit": the categorical *product* is given by a subset of the product of the underlying sets with δ given by the $\wedge$ of the δs of the coordinates. The obvious way to perturb this definition is to swap the $\wedge$ for a $\otimes$ – which works. We, however, found a wealth of ways of altering the construction.

By "wealth of ways" we mean appropriate congruences on locale of idempotent elements of $\mathscr{Q}$ induce each a monoidal structure on the category – and congruence inclusion induces monic "anti lax" monoidal functors (which are akin to oplax and lax monoidal functors but the relevant arrows

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[1]The quantales that enjoy *both* idempotency and integrality are locales with $\wedge$ as their monoidal product, and all locales enjoy those properties.

go in opposing directions)[2] between the resulting monoidal categories (where the underlying category remains the same) in a natural way.

Equality – the smallest congruence – in fact induces the obvious perturbation of the definition we mentioned; we have also found that the chaotic/trivial congruence – $\forall a, b : a \sim b$ – always induces a monoidal *closed* category (i.e., there exists a associated exponential, a kind of "space of functions") – a relevant additional structure for the purposes of categorical logic – in and it's *never* (even when $\mathscr{Q}$ is a locale) the usual product (because any non-equality relation cannot induce products with projections). The specific case of this when $\mathscr{Q}$ is a locale is – as far as we know – a novel construction in the field of $\mathbb{H}$-sets.

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[2]This is a novel notion as well, as far as we know. It is not clear to us if there are more examples of this anti lax functors "in the wild", but in this instance it arose rather naturally.

A Coq formalization of Reo connectors for cyber-physical systems

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Abstract

We can understand a Smart City as a group of devices like sensors, cameras, and semaphores, that communicate with each other and are coordinated to achieve functionalities to improve the life of the people who live in it, creating a way for this coordination to work is a real challenge and one of the main obstacles in achieving the concept of a Smart City. Reo [1], a graphical-based exogenous coordination language based on channels can be used to model this communication and coordination of isolated devices in a city.

Reo was created to model the integration of independent software components without having to know the details of each component, focusing on the structure and the behavior of the connectors that the components use to interact. Channels in Reo have two ends, which can be of two types: a source end, which accepts data into the channel, and a sink end where data flows out of the channel. Figure 1 shows a basic set of channels [2] that are used to compose more complex connectors.

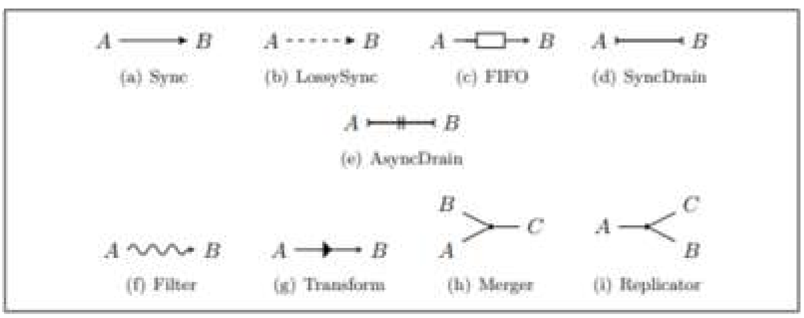

Figure 1: Basic set of channels

Another challenge in achieving the goal of a smart city is the criticality of its systems, meaning that the cost of failure of these systems is too high, therefore, software with high reliability is needed. This reliability can be achieved with formal verification of the software that can be done in a proof assistant characterized as an environment that enables the proof of properties of the objects formalized in it. In order to formalize a Reo model in such environment, a formal semantic is needed, since Reo is only a graphical formalism. *Constraint Automata* [2] is one of the formal semantics normally used to formalize Reo, it is an automata model where you have an automaton defined for each Reo connector. Firstly, states represent the possible configurations of a channel, and transitions describe how data in the connector flow, the latter already has its implementation [4] in Coq proof assistant done in other projects.

The problem with using *Constraint Automata* in a Smart City context is the inability to represent continuous situations that its devices, which are examples of cyber-physical systems, require. A cyber-physical system is the integration of computational systems and sensors and actuators that interfere with the physical environment and, consequently, resulting in continuous data.

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Constraint Automata is not sufficient, for example, to formalize a connector to model the delay in the flow of information in a parameterized amount of time, but it could be tried in such way:

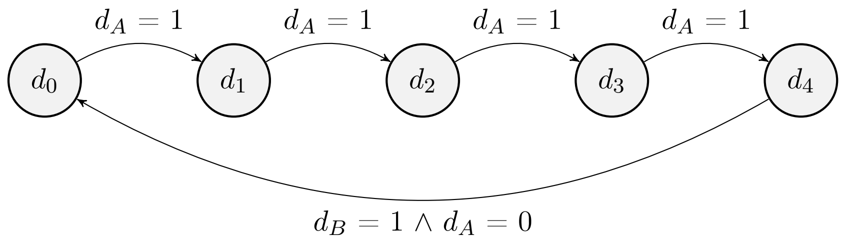

Figure 2: Delay conector in CA

The idea behind this connector is that when data arrives at the first port it needs to wait 4 transitions before flowing to the next port ($d_A = 1$ and $d_B = 1$ depict the presence of data in ports A and B respectively), but this does not ensure the delay of a predetermined amount of time (e.g 4 seconds).

Hybrid Constraint Automata [3] is a formal semantic more suitable to model these types of continuos behavior. It uses the bases of *Constraint Automata* but each state of the automaton has a Dynamical System associated with it. These dynamical systems can be thought of as a point in R^n space that has its trajectory governed by a function and a *Space Constraint*, a condition that the point can satisfy so a transition can be fired, the example of the delay connector above can be defined in HCA as:

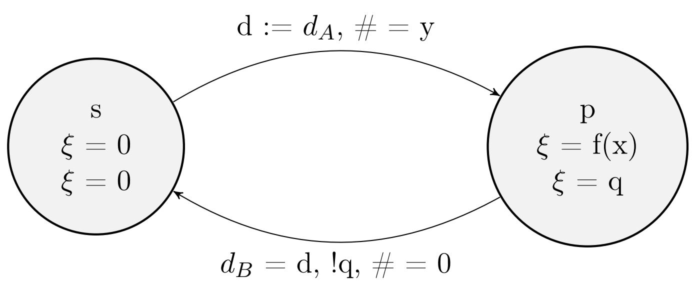

Figure 3: Delay conector in HCA

The automata execution begins in a state s, once data is received in port A it transitions to state p that has a dynamical system to model the decrease of a parameterized value and when it reaches 0 the transition is fired, data is accepted to port B and it returns to state s.

What has been done in this project is the implementation of *Hybrid Constraint Automata* to model Reo in Coq proof assistant and the creation of a compiler to generate certified Reo code with ease. For the formalization of HCA in Coq, it was used as a base the implementation [4] of CA done in other projects with the adaptations needed to simulate HCA.

Firstly for the adaptation, it was necessary to formalize the dynamical system, which is defined as a tuple (I, F, V, SC) with I as the initial value of the point, F as the trajectory function, V as the current value of the point, and SC as the Space Constraint. Another main adaptation that was needed to simulate an HCA was in the function that decides which states are accessible at each moment in the run, for a transition to be available to be used in HCA it needs to satisfy the conditions imposed for transitions in CA with the addition of the condition imposed by the dynamical system, meaning that not only data flow can trigger a transition but also the dynamical system configuration, that change is represented by this code snippet.

```
Fixpoint step' (theta : set tds) (portNames: set name)
(transitions: set(set name × DC name data × state))
(statesList: list state) (DSList: list (nat × (nat → nat) × nat × nat)): set state :=
match transitions with
| [] ⇒ []
| a::t ⇒ if
(set_eq (portNames)((fst(fst(a))))) &&
(evalCompositeDc (theta) (snd(fst(a)))) &&
(evalSC (snd(fst(DSBind (snd(a)) statesList DSList))) (snd(DSBind (snd(a)) statesList DSList)))
then snd(a)::step' theta portNames t statesList DSList
else step' theta portNames t statesList DSList
end.
```

The transitions which are accessible in a given state are defined by satisfying these 3 conditions separated by "&&" above, the last one checks if the dynamical system current value is equal to the dynamical system constraint value for this state. The other addition was the creation of the functions that handle the update of the dynamical system.

With this implementation is now possible to model connectors like the delay mentioned above in Coq, and prove properties about those connectors that are expressive enough to represent scenarios not only with discrete processes but continuos as well. Thanks to the way Coq proofs are built, using tactics, defined processes that manipulate the proof state and the building blocks for proofs, its extraction mechanism allows the transformation of formalized proofs and functions into functional programs in higher-level languages such as Haskell in an automatic manner, producing certified Reo programs that maintain the properties which were proven in Coq.

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Contradictions without gluts

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Abstract

Dialetheism is the view that some contradictions are true (i.e. the so-called 'dialetheias'). From this perspective, true contradictions embody *truth-value gluts*, that is, they are both true and false (see [3], [4] for the canonical presentations). However, even according to dialetheists' standards, dialetheias are rare, most contradictions being actually false. That works in particular for all known contradictions appearing in empirical sciences so far [4, chap. 9]. Still, according to dialetheists, when a context is consistent, we should use classical logic; when a context involves a contradiction, we should adopt a paraconsistent system (more specifically, LP, the Logic of Paradox, or some variation of it). This description of what to do when facing a contradiction, we claim, will impose a kind of *double standard*, and there comes the problem in the dialetheist setting: what allows for paraconsistency to obtain is precisely the presence of gluts. That means that paraconsistency is allowed for only in the rare contexts involving gluts (see [4, p.83]). But then how can we deal with contradictions appearing in inconsistent non-glutty contexts, such as those furnished by inconsistent science? On the one hand, such contexts appear to involve contradictions, and according to the above recommendation, we are told to use a paraconsistent logic. However, not being actually *true* contradictions, i.e., not being gluts, paraconsistency has no place here, and it is classical logic which is actually demanded, according to the other side of the same recommendation seen before [3, chap.8]. On the other hand, treating the context with classical logic simply gives us no resources for deductively dealing with the contradiction without triviality. Certainly, one could appeal in this case to systems of paraconsistency dealing with contradictions of a more epistemic nature, such as those proposed by Carnielli and Rodrigues [2], but by doing so one immediately loses the gluts, and become non-dialetheist; that is, by choosing this option all of the motivations for gluts coming from the treatment of semantic paradoxes are lost, and the epistemic position does not have resources to deal with that; see [1]. Dually, as we shall discuss, the dialetheist faces trouble dealing with such non-semantic contradictions, i.e., those contradictions that do not embody gluts, and have a more 'epistemic' nature (and recall again that, among contradictions, just some few are true). More precisely put: what are dialetheists going to do with contradictions in contexts that do not involve gluts? The dialetheist seems to be able to deal only with contradictions that are gluts, and has no plausible story to tell in such cases, just as the epistemic approach by Carnielli an Rodrigues had the resources to deal with epistemic contradictions, but not with gluts. In this talk, we not only present in a clear way the kind of difficulty that dialetheists face in dealing with contradictions without gluts, but also evaluate some of the options for a way out, discussing their plausibility in the context of a dialetheist theory.

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Change of logic, without change of meaning

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Abstract

Despite the mundane fact that logicians disagree, there are eminent arguments attempting to promote the claim that such disagreements are not *genuine*. Perhaps the most important argument in this direction is attributed to Quine, and is known as the *Meaning-variance argument*. The context where the argument is presented concerns a discussion between a logician who denies the validity of the law of non-contradiction, and another one, who attempts to judge on the consequences of doing so:

> My view of the dialogue is that neither party knows what he is talking about. They think that they are talking about negation, '$\sim$', 'not'; but surely the notion ceased to be recognisable as negation when they took to regarding some conjunctions of the form '$p\ .{\sim}p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject. (Quine [3, p.81])

Traditionally, the passage is considered as advancing the claim that when two parties connect different logical laws and/or inferences with a given connective, they are no longer attributing the same meaning to the connective, and, as a result, are no longer holding different views about the same thing. As Warren puts it:

> ...the best explanation of this meaning change is that one or more of the logical constants occurring in the sentence have changed their meaning. This thought can be spelled out in a number of roughly equivalent ways, but all of them involve the idea that, for example, meaning what the classical logician means by "not" and "or" *suffices* for acceptance of any instance of excluded middle whatsoever, at least potentially. So, if some particular instance of excluded middle isn't accepted, it must be because either "not" or "or" (or both) are being understood in a non-classical fashion. ([5, p.423])

This way of putting the issue brings to the center of the stage the basic claim of the meaning-variance argument: different systems of logic attribute different meanings to the connectives — validating different laws and inferences as a result —, and this, the argument goes, eliminates the possibility of genuine rivalry; disputes are merely verbal.

This talk is a contribution to resist the transition from lack of common meaning to lack of rivalry. We do this by offering a general *semantic* framework which, we shall argue, brings a common semantic basis to important systems differing on their laws and valid inferences. This points to a simple fact: when it comes to some systems of logic — we focus here on classical logic (**CL**), the Strong Kleene logic (**K3**) and the Logic of Paradox (**LP**) —, there may be a common basis in which to frame the disagreement, a basis in which meaning variance for the connectives is absent, although the systems may differ in which laws and inferences they validate. We then discuss how the framework bears on the original meaning-variance claim, as typically attributed to Quine. Additionally, we argue that the framework is able to profitably address yet another, more ambitious claim concerning meaning variance, which was advanced by Slater [4]. Finally, we shall comment on how our framework relates to similar approaches already available in the

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literature also attempting to resist meaning variance. We deal in particular with the proposals of Paoli [2] and Hjortland [1]. We conclude with further considerations about how to situate the kind of semantics we have advanced in the wider context of the debate about meaning variance.

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Algebraizability as an algebraic structure

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Abstract

By *logic* we mean a pair $(\Sigma, \vdash)$ where Σ is a signature, i.e. a collection of connectives with finite arities, and $\vdash$ is an idempotent, increasing, monotone, finitary and structural consequence relation on the set $Fm_\Sigma(X)$ of formulas over Σ with a set X of variables. A *translation* between logics $(\Sigma, \vdash) \to (\Sigma', \Vdash)$ is a map $f\colon Fm_\Sigma(X) \to Fm_{\Sigma'}(X)$ induced by an arity preserving map $\Sigma \to Fm_{\Sigma'}(X)$. It is called *conservative* if $\gamma \vdash \varphi \iff f(\Gamma) \vdash' f(\varphi)$.

Remote Algebraizability

A *remote algebraization* of a logic L is a jointly conservative family of translations $f_i\colon L = (\Sigma, \vdash) \to (\Sigma_i, \vdash_i) = L_i$ to algebraizable logics L_i. Remote algebraization has been introduced by Bueno et al. in [1] and successfully applied to non-algebraizable, and generally badly behaved, paraconsistent logics, providing a semantical meaning and algebraic contents.

Recall that a logic is algebraizable if it has a set Δ of equivalence formulas and a set $\langle \delta, \epsilon \rangle$ of pairs of formulas satisfying certain syntactic conditions given in [3, Thm. 4.7].

Definition 1. *A logic is called (n, m)-algebraizable, if it admits an algebraizing pair $(\Delta, \langle \delta, \epsilon \rangle)$ for which Δ consists of at most n formulas and $\langle \delta, \epsilon \rangle$ consists of at most m pairs of formulas.*

The following construction forces a logic to become (n, m)-algebraizable:

Definition 2. *Given a logic $L = (\Sigma, \vdash)$, one defines the logic $L \otimes A_{n,m} = (\Sigma', \vdash')$ as follows:*

Σ' is obtained by adjoining binary connectives $\Delta_1, \ldots, \Delta_n$ and unary connectives $\delta_1, \ldots, \delta_m$, $\epsilon_1, \ldots, \epsilon_m$ to the signature Σ. We abbreviate $\Delta = \{\Delta_1, \ldots, \Delta_n\}$ and $\langle \delta, \epsilon \rangle = \{\langle \delta_1, \epsilon_1 \rangle, \ldots, \langle \delta_m, \epsilon_m \rangle\}$.

$\vdash'$ is the consequence relation generated by the rules of $\vdash$ and the rules making $(\Delta, \langle \delta, \epsilon \rangle)$ into an algebraizing pair.

Clearly we have an inclusion $L \to L \otimes A_{n,m}$ which is a translation, and this is a generic candidate for a remote algebraization.

Proposition 3. *A logic L admits a remote algebraization by a finite family of translations if and only if the translation $L \to L \otimes A_{n,m}$ is conservative for some $n, m \in \mathbb{N}$.*

Using this equivalence, we can characterize the finitely remotely algebraizable logics:

Theorem 4. *A logic $L = (\Sigma, \vdash)$ is remotely algebraizable by a finite family of translations if and only if one of the following conditions holds:*

(1) L has theorems.

(2) L admits no derivation of the form $\{x\} \vdash \varphi$ in which the variable x does not occur in φ.

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The theorem and its proof also elucidate what are the possible obstructions to the algebraizability of a logic: On the one hand it can be missing connectives for forming an algebraizing pair – this is what we try to remedy with the construction of Def. 2. On the other hand it can be a kind of explosive behaviour, excluded by condition (2), which even prevents adding such connectives in a conservative manner!

Algebraizability as algebraic structure

We consider the category **HoLog** whose objects are logics and whose morphisms are equivalence classes of translations, where translations f, g are equivalent iff $f(\varphi) \dashv\vdash g(\varphi)$ for all φ in the domain.

The construction $L \mapsto L \otimes A_{n,m}$ of Def. 2 is part of a functor $\mathfrak{A}_{n,m}\colon \mathbf{HoLog} \to \mathbf{HoLog}$. We have natural transformations $\mathrm{id} \to \mathfrak{A}_{n,m}$ given by the inclusions of Prop. 3 and $\mathfrak{A}_{n,m} \circ \mathfrak{A}_{n,m} \to \mathfrak{A}_{n,m}$ given by identifying the two copies of formulas of the algebraizing pair.

Theorem 5. *(1) The functor $\mathfrak{A}_{n,m}$ with these two natural transformations is a finitary monad on* **HoLog***. (2) A logic is (n,m)-algebraizable if and only if it admits an algebra structure for the monad $\mathfrak{A}_{n,m}$ (3) A logic admits at most one $\mathfrak{A}_{n,m}$-algebra structure. (4) The category of $\mathfrak{A}_{n,m}$-algebras is equivalent to the category of (n,m)-algebraizable logics and morphisms that preserve algebraizing pairs.*

From previous results of the authors one can derive that **HoLog** is locally finitely presentable. Results on monads and accessible categories then yield the following consequences:

Theorem 6. *(1) The category of (n,m)-algebraizable logics, and equivalence classes of algebraizing pair preserving translations is locally finitely presentable.*

(2) The category **HoAlg** *of algebraizable logics, and equivalence classes of algebraizing pair preserving translations is accessible.*

In particular the categories of (n,m)-algebraizable logics are equivalent to categories of models of finite limit theories, and the category **HoAlg** is equivalent to a category of models of an infinitary first order theory. This is a priori not at all clear, given the several places in which the definitions of logics and algebraizable logics refer to subsets.

Other Leibniz classes

The setup of a filtered collection of logics like the (n,m)-algebraizable logics above is precisely mirrored in Jansana's and Moraschini's definition of Leibniz class [2]. In the final part of the talk we discuss how much of the above results extend to general Leibniz classes. For example for protoalgebraic logics, everything up to Thm. 5(1) and (2) goes through, but since the implication formulas witnessing protoalgebraicity are not unique, as an analog of Thm. 5(4) we obtain we obtain an equivalence with the category of protoalgebraic logics *with a chosen set of witnessing formulas.*

The analog of the construction of Def. 2 is actually a *coproduct* with a generic protoalgebraic logic, and this allows for a descent theory by which one can detect whether a logic is protoalgebraic to begin with. This is in contrast with algebraizable logics, where the construction is not a coproduct and where no such detection mechanism exists.

We finish by sketching an emerging general theory of monads and descent for Leibniz classes.

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Unfriendly partitions when avoiding vertices of finite degree

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Abstract

The Unfriendly Partition Conjecture is a well known open problem toward infinite graphs and important studies in this subject lie in the intersection between Combinatorics and Set Theory. Most of its popularity is probably due to its simplicity as a statement. In order to introduce it, we say that any function $c : V \to 2$ is a *coloring* of a graph $G = (V, E)$. Regarding the natural partition described by c, we say that two adjacent vertices $u, v \in V$ are *friends* if $c(u) = c(v)$, and *enemies* otherwise. Denoting by $N(v)$ the neighborhood of a vertex $v \in V$ and by $d(v) = |N(v)|$ its degree, we say that the coloring c is *unfriendly in* v if this vertex has no less enemies than friends, namely, if $|\{u \in N(v) : c(u) \neq c(v)\}| \geq |\{u \in N(v) : c(u) = c(v)\}|$. In particular, if v has infinite degree, there is a simpler way to state when c is unfriendly in v: it is necessary and sufficient that v has $d(v)$ enemies. Globally, c is said to be *unfriendly* if it is unfriendly in every vertex of G.

By considering a coloring that maximizes the amount of edges with distinct colors in its ends, it is easily verified that every finite graph has an unfriendly partition. Inspired by this observation, Cowan and Emerson in an unpublished paper [1] conjectured that every graph admits an unfriendly partition. While this is still unknown for countable graphs, Milner and Shelah exhibited in [2] a family of uncountable graphs that cannot be partitioned that way. Besides being the only known graphs in the literature that have no unfriendly partitions, they have some other particularities. First, even though it is easy to check whether a coloring is unfriendly in a vertex with infinitely many neighbors, those graphs have no vertices of finite degree. Second, the least of them has $(2^\omega)^{+\omega}$ vertices, where $(2^\omega)^{+\omega}$ denotes the first limit cardinal greater than the continuum.

Although this seems to be an excessive amount of vertices, in this presentation we argue that the construction of Milner and Shelah is somehow minimal. More precisely, let κ be the least cardinal for which there is a graph with κ vertices, all them of infinite degree, that has no unfriendly partition. We show that the equality $\kappa = (2^\omega)^{+\omega}$ is independent from ZFC. Actually, under the Continuum Hypothesis (CH), its consistency is easily obtained by previous results toward unfriendly partitions due to Aharoni, Milner and Prikry in [3]. In ZFC+CH, however, the cardinals $\aleph_\omega$ and $(2^\omega)^{+\omega}$ are the same. We then improve this independence result by showing that, in ZFC+$\aleph_\omega < (2^\omega)^{+\omega}$, a theory where $\aleph_\omega \neq (2^\omega)^{+\omega}$, there are two possible values for κ: both statements $\kappa = \aleph_\omega$ and $\kappa = (2^\omega)^{+\omega}$ are independent.

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Legal Gaps

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Abstract

There are two situations in a Juridical System that generates hundreds of years old discussions. The first one is as follows: a child is drowning in a lake and a passerby is seeing the drama. He has to react accordingly: Ought he to save the child? Yes: there is an obligation to save someone in mortal danger, otherwise one is accused of failure to render assistance to a person in danger. No: it is forbidden to swim in this lake, according to the local *Juridical Norm*. This means that the passerby is both obliged and forbidden to enter the lake in order to save the child? The other situation is subtly different. Like the first situation, there is a child drowning and a passerby. This time, however, there isn't any legal obligation, neither to save a person in mortal danger nor to refrain from swimming in the lake. In this case, what is to be done from the juridical logic perspective? Anything goes, or is there some kind of constraint? The first case is a classical example of an *antinomy*, which correlates to legal *inconsistency*, while the second is an example of a legal gap, which corresponds to what is called legal *incompleteness*. Our questioning is twofold: What does *legal gap* mean? And how to deal with it from a philosophical and logical perspective? The first perspective is the *positivist* approach stating that there are no *real legal gaps* but only *ideological gaps*, insofar as Law is understood as a Juridical System that is taken to be consistent and complete. The second perspective is the *rhetorical* approach stating that there exists real legal gaps, but they are all solvable only by rhetorical means since Law is not a logical system, but an *almost-logic* one. The third perspective states that gap is to *paracomplete* juridical reasoning what legal *glut* is to *paraconsistent* juridical reasoning. Our point is that there are at least three juridical answers in front of this state of affairs. Either there is always a criteria (e.g. hierarchical, chronological, specialty, competency etc.) that decides which Juridical Norm should be used to give only one deontic normative value for an action, i.e., the passerby is either obliged or forbidden to save the child. Or there is no such criteria, by which a conflict between two different deontic normative values arises. This conflict is one of the following two: the passerby is both obliged and forbidden or the passerby is neither obliged nor forbidden to save the child in the eyes of general Law. What of this legal *pluralism*? A many-valued system of juridical logic is proposed to account for a number of problems related to philosophy of law, especially the case of legal gaps. While a number of papers have been devoted to the case of paraconsistent legal logic (thorough the issues of inconsistent data bases and *defeasible rea- soning*), the following wants to stress on legal gap as a case for paracomplete juridical reasoning. We propose a general framework for this purpose: $\mathbf{AR}_{4_L}$, which is a 4-valued juridical system including the aforementioned Juridical Systems as particular sublogics. In the vein of Von Wright's truth-logic, it consists of a formal language of *Juridical Statements* Sp, to be read 'There is a Juridical Norm that states (the action described by the sentence) p' (where the action expressed by p is not indifferent in the eye of the law). Then negation may be prefixed to either S or p, leading to a set of 16 possible basic formulas among the 4 basic ones: Sp, $\overline{S}p$, $S\overline{p}$, and $\overline{S}\overline{p}$. A deontic interpretation of S depends upon which kind of Juridical System is mentioned, with its correspondent *rule of legal closure*, whether it be Common Law ('If something is not prohibited then it is permitted") or Civil Law ('If something is not permitted then it is prohibited"). In the former, a *promulgation* entails that doing what a Juridical Norm states is permitted; whereas in the latter, a promulgation entails that doing what a Juridical Norm states

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is forbidden. Permission will be viewed as the basic deontic normative value, in the following. A semantics for Juridical Statements is an interpretation assigning deontic normative values to statements p, and juridical pluralism stems from the plurality of assignment conditions of them: (1) There is a promulgation p, and there is a promulgation $\overline{p}$; (2) There is a promulgation p, and there is no promulgation $\overline{p}$; (3) There is no promulgation p, and there is a promulgation $\overline{p}$; (4) There is no promulgation p, and there is no promulgation $\overline{p}$. (1) and (4) correspond to legal gluts and gaps, and we will defend a many-valued treatment of these after a survey of the relevant literature in legal logic.

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Undecidability of indecomposable polynomial rings

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Abstract

We assume that all our rings are commutative and unital. A ring is said to be *indecomposable* if its only idempotent elements are 0 and 1 (equivalently, if it cannot be written as the direct product of two nonzero rings), and *reduced* if 0 is its only nilpotent element. For example, every integral domain is reduced and indecomposable (but the converse does not hold).

We initially consider reduced indecomposable polynomial rings (in an arbitrary set of indeterminates) and further explore the constructions made in [1] (which deals with definability of integers in such rings) to attain two related goals, namely: undecidability of their first-order theory, and interpretability of arithmetic. We restrict ourselves to the one-variable case and show that the general case follows from it.

For the first goal, we extend the scope of a method by Raphael Robinson [2] to prove undecidability of a given ring. Such a technique relies on the existence of suitable, distinguished elements in the ring whose sets of powers are definable in a parametric way, and uses the powers of such elements and the parametric formula to codify the addition and multiplication of natural numbers.

Robinson is able to apply his technique in the case of polynomial rings with coefficients in a field or in the ring of $\mathbb{Z}$-integral elements of a number field. Our first contribution is the construction of two formulas, one for the set of suitable elements and another for the parametric definition of the sets of powers: these formulas work in a uniform way across all rings in the class considered, which also includes all polynomial integral domains. Notice that the definability result of [1] implies undecidability of the rings only in the case of characteristic zero; in contrast, this proof of undecidability is irrespective of the characteristic or any other specific properties of the rings.

Regarding interpretability, the drawback of Robinson's method is that it works indirectly (though in a very clever way) with the exponents of the powers of the distinguished elements of the ring. In particular, it does not resort to any codification of the condition of equality of such exponents, and for this reason such a method does not qualify, by modern standards, as an *interpretation* of arithmetic in the ring.

In this direction, we refine the constructions made in the proof of undecidability in order to construct formulas for interpretation of equality of exponents, at the cost of having to consider three separate cases for the coefficient ring: a nonfield, a field of characteristic zero, and a field of positive characteristic.

In the first case we study the algebraic condition "$a - b \mid a^m - b^n \implies m = n$", and we put it to good use for interpreting equality of exponents. Afterwards, we use Robinson's codification for addition and multiplication of natural numbers in order to interpret these operations in this case. As in our undecidability result, this construction is irrespective of the characteristic (again, our previous definability result [1] provides a trivial interpretation of arithmetic in characteristic zero).

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When the coefficient ring is a field, the method above no longer works. Moreover, the case of fields of characteristic zero is already treated in [2], where a formula defining $\mathbb{N}$ uniformly in polynomial rings (in any set of indeterminates) over a field is given.

Thus, it remains to consider the case of the coefficient ring being a field of characteristic $p > 0$. In this case we restrict our attention to the exponents of the form p^m, and exploit the properties of the Frobenius endomorphism to construct interpretations of equality of such exponents. Having this, we tweak Robinson's codifications of addition and multiplication in order to interpret sum and product between the exponents of the powers of p involved (not the powers themselves).

In the final part of our work we remove the reducedness hypothesis in our results. To this end, we exploit the uniform definability of the nilradical in arbitrary polynomial rings (it is equal, in these rings, to the first-order version of the so-called *Jacobson radical*), and the fact that an indecomposable polynomial ring becomes reduced and remains indecomposable and polynomial after factoring by its nilradical. This, together with the fact that rings modulo a definable ideal are interpretable in the given ring (via the projection homomorphism), allows us to transport our undecidability and interpretability results, valid so far only for reduced indecomposable polynomial rings, to the larger class of indecomposable polynomial rings.

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On validity paradoxes and (some of) their solutions

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Abstract

The naïve predicate of validity is widely investigated in philosophical logic because it is a fundamental concept to reasoning. However, it is a well-known fact that this notion is as problematic as the naïve truth predicate since naïve validity is also subjected to inconsistency results when it extends formal languages containing arithmetic. Such inconsistency results are known as Montague's Theorem [6] and validity-Curry [1]. The non-classical solutions to the validity paradoxes are usually preferred over the classical ones because the former block the problematic steps in the derivations of both paradoxes while they allegedly preserve the intuitive aspects of naïve validity. In this talk, I will compare some classical Skyrms [8], Ketland [5], Barrio et al [2], Field [3], Hlöbil [4], Wansing and Priest [9] and Pailos [7] to validity paradoxes and argue that it is not philosophically immediate that non-classical solutions are preferable to the classical ones. Even if those solutions do not exhaust all the solutions found in the literature, they somehow raise more general questions about the solutions to the paradoxes of naïve validity.

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Interpreting ZFC With A Librationist Set Theory

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Abstract

Revisions to the librationist set theory of [1], now dubbed £ ("pound"), are presented, so that the recast set theory Ꝉ ("libra") suffices to interpret classical set theory *ZFC*. The more pristine formal language of Ꝉ is in a Polish manner, without the identity symbol, and, as with [3], and in much literature on non-classical set theories, including so-called *property theories*, as [2 & 4], and others, abstracts are used, for the principle of extensionality fails. One should bear in mind that many results of [1] are not superseded.

1 Definition a) Long equality sign $==$ belongs to the metalanguage, as do & for conjunction, $\Rightarrow$ for implication, $\Leftrightarrow$ for bi-implication, Σ for existential quantification and Π for universal quantification; b) $\bullet$, v, $\downarrow$, $\forall$ and $\odot$ are the *symbols*, and denote the natural numbers 1, 2, 3, 4, and 5; c) Concatenation $\frown$ is the number theoretic function such that $m \frown n$ is $m \cdot 5^{\ell(n)} + n$, where $\ell(n) == \lfloor log_5((n+1)\cdot(5-1))\rfloor$ invokes the floor function $\lfloor\ \rfloor$ and defines the length of the numeral needed to express the positive natural number n in the bijective base-5 numeral system; d) We often write mn for $m \frown n$. e) v is a variable; f) A variable succeeded by $\cdot$ is a variable; g) Nothing else is a variable; h) Variables are terms; i) q, r, s, t, u range over variables, a, b, c, d, e range over terms and A, B, C, D, E range over formulas; j) If a and b are terms, ba is a formula; k) If A and B are formulas then $\downarrow AB$ is a formula; l) If A is a formula and u is a variable, then $\forall uA$ is a formula; m) If A is a formula and u is a variable, then $\circ uA$ is a term; n) Nothing else is a term or a formula; o) In $\forall uA$, u is the *bind* of A and the *tie* of $\forall$. A is the *scope* of $\forall$; p) In $\circ uA$, u is the *bind* of A and the *tie* of $\circ$. A is the *scope* of $\circ$; q) An occurrence of a variable in a formula (term) is bound, just if a bind, or in the scope of a binder with another occurrence as tie; r) Occurrences of variables in formulas (terms) are *free* if not bound; s) Variables are free (bound) in formulas (terms) just if an ccurrence is; t) A formula with no free variables is a *sentence*; u) A term with no free variables is a *constant*.

2 Definition a) $\neg A == \downarrow AA$; b) $A \to B == \downarrow\downarrow\downarrow AAB \downarrow\downarrow ABB$; c) $A \wedge B == \downarrow\downarrow AA \downarrow BB$ d) $A \vee B == \downarrow\downarrow AA \downarrow BB$; e) $\exists uA == \downarrow \forall uA \forall uA$; f) $b \in a == ba$; g) $\{u|A\} == \odot uA$

3 Definition Austerity is relaxed further by using parentheses for punctuation.

4 Definition The notion of *thesishood* is in the sense of Definition 16, and we do not make an effort towards a partial axiomatization of the mentioned theses here.

5 Definition We distinguish between two types of theses, while appealing to Definition 16:

(1) $\overset{M}{\models} A \overset{df}{=} \models A \ \&\ \not\models \neg A$ *maxim*, or *maximal thesis*

(2) $\underset{m}{\models} A \overset{df}{=} \models A \ \&\ \models \neg A$ *minor*, or *minor thesis*

6 Fact The *maxim mode*, which says B is a maxim, if A and $A \to B$ are maxims, is valid in £ and Ꝉ. Thirteen more inference modes can be lifted from [1, 139–140]. Modus ponens for $\models$ is not valid.

7 Definition $a = b == \forall u(a \in u \to b \in u)$ Comment: In a lasting contribution, [13] obtains an improvement upon *Leibniz' Law*, as the bi-conditional corresponding with Definition 7 is established in Principa Mathematica's theorem *13.101, on account of its predicative Definition *13.1 and its *Axiom of Reducibility* *12.1. Relation $=$ of Definition 7 is obviously reflexive and transitive. On account of the liftable Lemma 1 of [1,342], it is as well a maxim that $a \in \{r|\forall s(r \in s \to a \in s)\}$, so the relation is symmetric. Moreover, it is a maxim that $\forall r, s(r = s \to (A \to A^r_s))$, where A^r_s is the result of substituting all free occurrences of s in A with r.

8 Definition a)$\{a\} == \{x|x = a\}$;b)$\emptyset == \{x|x \neq x\}$;c)$\omega == \{x|\forall y(\emptyset \in y \wedge \forall z(z \in y \to \{z\} \in$

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$y) \to x \in y)\}$

9 Observation Our number theoretic point of view is slightly stronger than [8], as it presupposes that the expressions of the formal lannguage directly denote natural numbers.

10 Fact All conditions define a set, so for any formula A, it is a maxim that $\exists u(u = \{t|A\})$.

11 Definition $\langle a, b\rangle$ is the ordered pair, taken e.g. à la Kuratowski.

12 Theorem If $A(r, s)$ has the variables shown, for an a: $\overset{M}{\models} \forall u(u \in a \leftrightarrow u \in \{t|t \in \{s|A(s, a)\}\})$

Proof: The construction, of what we call *manifestation points*, goes back to [2,78]; related results were obtained by many others: Let a be $\{r|\langle r, b\rangle \in b\}$ where b is $\{\langle s, t\rangle|A(s, \{u|\langle u, t\rangle \in t\})\}$. □

13 Definition Let $\overset{M}{\models} \forall u(u \in \Gamma \leftrightarrow u \in \{t|t \in \{s|(s \subset \{r|r \in s\}) \wedge (s \subset \Gamma)\}\}\})$

14 Definition A^a signifies that all variables bound in A are bound to a, and $\{x|A\}^a$ is $\{x|x \in a \wedge A^a\}$

15 Definition Condition $C(x, y)$ is *extent-functional* $\overset{df}{=} (C(x, y) \wedge C(x, z) \to \forall w(w \in y \leftrightarrow w \in z))$

Let b *contain* a just if $a \in b$, and let the following maximal axiomatic principles fill Γ:

(i) Γ contains $\{u|u \in u \wedge u \notin u\}^\Gamma$

(ii) Γ contains $\{a, b\}^\Gamma$ if it contains a^Γ and b^Γ

(iii) $\exists u[\forall x(\neg\exists y(y \in x) \to x \in u) \wedge \forall x\forall y(x \in u \wedge \forall z(z \in y \leftrightarrow \forall w(w \in z \to w \in x)) \to y \in u)]^\Gamma$

(iv) If Γ contains a^Γ, it contains $\{x|\exists y(x \in y \wedge y \in a)\}^\Gamma$

(v) Γ contains $\{x|x \subset a\}^\Gamma$ if it contains a^Γ

(vi) $a \in \Gamma$ only if $(\exists y(y \in a) \to \exists y(y \in a \wedge \neg\exists z(z \in a \wedge z \in y)))^\Gamma$ (Foundation)

(vii) If Γ contains a^Γ, and $C(x, y)^\Gamma$ is an extent-functional first order condition upon x and y, then Γ contains $\{y|\exists x(x \in a \wedge C(x, y))\}^\Gamma$ (Replacement)

The strong version (iii) of infinity is from [10, 117], to conform with its construction to support the conclusion in the antepenultimate paragraph, as it depends upon the interpretative power of the system presupposed.

As we relate some more on in the antepenultimate paragraph, $\mathfrak{T}$, via Γ, interprets a version ZF^- of Zermelo-Fraenkel set theory minus extensionality.

The semantics of $\mathfrak{T}$ is by a set theoretic Herzberger revisionary process on ordinals, adapted from [1]. Let Ξ be a function from ordinal numbers to real numbers, i.e. sets of natural numbers, which, given the number theoretic point of view, are maximal consistent sets of formulas in the language of $\mathfrak{T}$, such that for all ordinal numbers α and β:

I $\Xi(\alpha) \Vdash$ (i) & (ii) & (iii) & (iv) & (v) & (vi) & (vii)

II $\Xi(\alpha) \Vdash\downarrow AB$ just if neither $\Xi(\gamma) \Vdash A$ nor $\Xi(\gamma) \Vdash B$

III $\Xi(\alpha) \Vdash \forall x A(x)$ just if $\Xi(\gamma) \Vdash A(a/x)$ for all a substitutable for x in A

IV $\alpha \prec \beta \Rightarrow (\Xi(\beta) \Vdash u \in \{v|A\} \Leftrightarrow \Sigma\gamma(\gamma \prec \beta \;\&\; \Pi\delta(\gamma \preceq \delta \prec \beta \Rightarrow \Xi(\delta) \Vdash A^u_v)))$

Let us call such a function as Ξ an *attractive* function.

16 Definition A formula A is *valid*, $\models A$, just if it for all attractive functions Ξ, at the *closure ordinal* ♀, of the Herzberger revisionary process, holds that $\Xi(♀) \Vdash u \in \{u|A\}$, or for all attractive functions Ξ, at the closure ordinal ♀, holds that $\Xi(♀) \Vdash u \notin \{u|\neg A\} \wedge u \notin \{u|A\}$.

17 Definition A formula A of $\mathfrak{T}$ is *paradoxical* just if both $\models A$ and $\models \neg A$, and a is paradoxical just if the formula $b \in a$ is paradoxical for some b.

18 Definition A formula A of $\mathfrak{T}$ is *orthodox* just if A is a maxim or $\neg A$ is a maxim, and a is orthodox just if $b \in a$ is always orthodox.

19 Fact Γ is a set of hereditarily orthodox sets

20 Assumption Γ is orthodox.

21 Fact There are distinct co-extensional orthodox sets, so the principle of extensionality must fail.

22 Fact Paradoxicalities, as Russell's sentence $\varrho \in \varrho$, for $\varrho = \{x|x \notin x\}$, are resolved in what we now call an *ultracohesive* manner. $\varrho \in \varrho$ and $\varrho \notin \varrho$ are valid in the semantics, and so minor theses.

23 Fact Unlike *paraconsistent* approaches, which only justify fragments of classical reasoning, $\mathfrak{T}$ *extends* classical logic, and does so *sedately* in the sense that if A is a thesis in $\mathfrak{T}$, then $\neg A$ is not a thesis in classical logic, and if A is a thesis in classical logic, then A is a maxim in $\mathfrak{T}$.

24 Observation $\mathfrak{T}$ avoids the counter intuitive categoricity property that for any formula A, either A is a thesis of £ or $\neg A$ is a thesis of £, resulting from the latter's fixation upon one model, as it only allows an empty beginning to the revisionary Herzberger process. In contrast, neither $\{x|x \in x\} \in \{x|x \in x\}$ nor $\{x|x \in x\} \notin \{x|x \in x\}$ is valid in $\mathfrak{T}$; notice that as a consequence of Definitions 17 and 18, $\{x|x \in x\}$ is neither paradoxical nor orthodox.

25 Fact If it is a maxim that a is extensionally distinct from the universal set $V \Longleftrightarrow \{u|u = u\}$,

then the unrestricted power set $\mathcal{P}(a) = \{x|x \subset a\}$ of a is paradoxical.
26 Assumption Let ω in Γ contain precisely, the finite von Neumann ordinals relativized to Γ.
27 Observation Suppose orthodox f surjects from ω to $\mathcal{P}(\omega)$. $S \Longrightarrow \{x|x \in \omega \wedge \langle x, x\rangle \notin f\}$ may be paradoxical, as $\mathcal{P}(\omega)$, per fact 25, and we cannot conclude that there is no surjection from ω to $\mathcal{P}(\omega)$.
Fact 28 Given Assumption 20, $\mathcal{P}(\omega)^\Gamma = \{x|x \in \Gamma \wedge x \subset \omega\}$ is orthodox.
28 Observation Suppose f surjects from ω to $\mathcal{P}(\omega)^\Gamma$, and $f \in \Gamma$. Given Fact 19 and Assumption 20, a contradiction follows, as with Cantor. So Γ has no function from ω onto $\mathcal{P}(\omega)^\Gamma$.
29 Observation Suppose an orthodox $f \notin \Gamma$ surjects from ω to $\mathcal{P}(\omega)^\Gamma$. We do not have the means to ascertain that $\{u|u \in \omega \wedge \langle u, u\rangle \notin f\} \in \mathcal{P}(\omega)^\Gamma$, so the Cantorian argument does not work.
30 Information Confer [1,346-351] for supplementary analyses of the situation.
31 Discussion Given these circumstances, it may be said that Γ *"erroneously believes"* that $\{x|x \in \Gamma \wedge x \subset \omega\}$ is not countable., which conforms with Skolem's resolution of the seeming contradiction between the fact that a classical set theoretic system "says" that there are uncountable sets, and the fact that the system has a countable model. But it does not need to support Skolem's conclusion, of [11, 12], that the notion of *set* is *relative*, as it is not presupposed that the surjection f is contained in a *classical* set theory.
31 Conclusion Above ZF^- was developed relative to Γ, so ZF^- is interpreted by $\mathfrak{B}$. [9, 130-131] established that ZF^- interprets ZF. As a consequence, $\mathfrak{B}$ interprets ZF, and it follows, given [5 & 6], that $\mathfrak{B}$ interprets classical *Zermelo Fraenkel set theory with the Axiom of Choice.*

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Dedução Natural para Lógica Linear com stoup

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Resumo

Originalmente proposta para introduzir a ideia de polaridade em um cálculo de sequentes para lógica clássica [1], a noção de *stoup* nos permite dividir o consequente de um sequente em duas regiões, uma região clássica e uma região intuicionista. Dado um sequente $\Gamma \Rightarrow \Delta; \Sigma$, sendo Γ, Δ e Σ *multisets*, dizemos que Δ é a região clássica (fórmulas negativas), enquanto Σ, cuja cardinalidade é restrita a no máximo uma fórmula, é a região intuicionista (ou *stoup*).

Inspirados no sistema para lógica ecumênica apresentado por Pereira e Pimentel [6], pretendemos construir um sistema de regras de dedução natural para lógica linear clássica proposicional com *stoup* (LNDs). Além das vantagens usuais que sistemas de dedução natural apresentam em relação a outros sistemas, dentre elas, a simplicidade na definição das constantes lógicas, a introdução do *stoup* se mostra bastante vantajosa, pois permite a formulação de um sistema puro[1] e harmônico[2] com *single-conclusion* para lógica linear proposicional. Outro aspecto a ser destacado de LDNs é a formulação para as regras dos exponenciais. Usando as ideias de Schroeder-Heister [8] e Nascimento [4], definimos as regras para o *bang* e para o *question mark* por meio da noção de regras abstratas.

Dentre os resultados que serão apresentados para LNDs estão a equivalência entre LNDs e o cálculo de sequentes para Lógica Linear de Girard, o que garante a corretude e a completude de LNDs, e o Teorema de Normalização via estratégia de Pottinger [7], que garante que toda derivação em LNDs possui uma forma normal.

Ao compararmos nosso sistema com outros sistemas encontrados na literatura, podemos encontrar certas vantagens. Por exemplo, LNDs define regras para FLL (*Full Linear Logic*), enquanto Medeiros [2] define regras apenas para os fragmento $\{\bot, \otimes, \multimap, !\}$. LNDs também possui vantagens em relação ao sistema de Negri [5] por ser capaz de tratar o caso clássico. LNDs também se mostrou um sistema mais *puro* que o sistema apresentado por Martins [3].

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[1] Uma regra de dedução natural para um operador α é dita puro quando a definição da regra não utiliza nenhum operador além de α. O conjunto de regras apresentados por Prawitz para Lógica Clássica, Intuicionista e Minimal possui apenas regras puras. Entretanto, algumas formalizações de sistemas para Lógica Linear, por exemplo [3], não possuem regras pura.

[2] Existem várias definições para a noção de harmonia. Neste trabalho, dizemos que um sistema é harmônico se satisfaz o princípio de inversão.

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Three-valued paraconsistent multimodalities and their meaning

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Abstract

This talk intends to present the main ideas of a recent paper to be published by Springer Synthese Library, entitled "Many-Valued Modalities and Paraconsistency" ([1]). The paper extends the three-valued paraconsistent logic **LFI1** (cf. [2]), a member of the family of Logics of Formal Inconsistency, to a class of multimodal systems, generating an infinite hierarchy of three-valued paraconsistent multimodal logics. Soundness and completeness in terms of possible-worlds (or Kripke semantics) are obtained by means of canonical models ruled by relation algebras, and a version of Dugundji's theorem is also obtained for **LFI1** enriched with primitive multimodal operators. Some aspects related to the interpretation and potential uses of such logics are discussed.

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Complexidade Descritiva de Processos Reversíveis em Grafos*

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Resumo

A Complexidade Descritiva [9] nos fornece uma perpectiva da Lógica para a área de Complexidade Computacional, em que as classes de complexidade são relacionadas com uma linguagem da lógica capaz de expressar os problemas das classes.

Neste trabalho, iremos utilizar as ferramentas da Complexidade Descritiva para apresentar uma análise de complexidade do problema conjunto conversor em processos irreversíveis e reversíveis em grafos [2,4][1], que consiste, de maneira simplificada, em determinar o menor conjunto de vértices "infectados" que, em uma sequência dinâmica "infecções" fazem com que todos os vértices fiquem "infectados". Dada a natureza iterativa do problema, nosso objetivo é expressá-lo por meio de lógicas com operadores de ponto-fixo como FO[LFP] e FO[PFP] (primeira-ordem com *menor ponto-fixo* e com *ponto-fixo parcial*, respectivamente).

Além disso, também iremos considerar a complexidade descritiva para a versão parametrizada do problema usando as caracterizações descritas em [3]. O problema parametrizado consiste em um problema de decisão com um parâmetro destacado além da entrada do problema.

Dado um grafo $G = (V, E)$ e uma função $f : V \to \mathbb{N}$ (denominada limiar), iremos considerar o processo iterativo em G dado pela seguinte regra: partindo de uma rotulação inicial dos vértices $c_0 : V \to \{0, 1\}$, cada vértices v irá mudar de rótulo se, e somente se, v tiver pelo menos $f(v)$ vértices adjacentes com o rótulo oposto. Um processo f*-reversível* em um grafo G consiste em sucessivas atualizações simultâneas dos rótulos de c_i, denominada configuração do processo no instante $i \in \mathbb{N}$, para c_{i+1} em que $i \geq 0$. Assim, um processo f-reversível em G a partir de c_0 descreve uma sequência $(c_0, c_1, \dots) = (c_t)_{t\in\mathbb{N}}$, e o denotamos por $P(G, f, c_0)$. Um processo é denominado *irreversível* se em algum momento um vértice v receber o rótulo 1, não torna a ser rotulado com 0. Quando $f(v) = \ell$ é uma constante, para todos os vértices de V, nós iremos denotar o processo ℓ-(ir)reversível por $P^{\leftrightarrow}_{\ell}(G, c_0)(P^{\rightarrow}_{\ell}(G, c_0))$.

Dado um processo ℓ-(ir)reversível em G, onde a função limiar é constante e igual a ℓ, o *problema do conjunto ℓ-conversor* consiste em determinar se existe um conjunto de vértices inicialmente rótulados com 1 (denominado conjunto conversor) e com cardinalidade menor que um limite k, que faz com que todos os vértices em V estejam rotulados com 1 em algum momento $t \in \mathbb{N}$. O problema é NP-completo para $\ell \geq 2$ [2,4] e W[2]-difícil [6] para grafos não direcionados.

Inicialmente, focaremos no caso particular do problema para um processo **2-irreversível** ($P^{\rightarrow}_{2}(G, c_0)$) em que desejamos saber se **um conjunto conversor de cardinalidade k também constante**, por exemplo $k = 2$, faz com que todos os vértices fiquem com rótulo 1 (observe que a função limiar e o tamanho do conjunto conversor são constantes). Para isso, nós construímos uma fórmula da lógica de primeira-ordem que fornece a atualização única dos vértices rotulados com 1,

$$\varphi(x, I)_{irr} := I(x) \vee \exists y_1 \exists y_2 (y_1 \neq y_2) \wedge I(y_1) \wedge I(y_2) \wedge E(y_1, x) \wedge E(y_2, x).^2$$

A partir de um conjunto inicial de vértices rotuldos $I \subseteq V$, a fórmula $\varphi(x, I)_{irr}$ define os vértices x que receberão rótulo 1 na próxima atualização. A fórmula $\varphi(x, I)_{irr}$ é positiva, não possuindo

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[1] Também apresentado na literatura como "Target-Set Selection problem" [6].

[2] A fórmula indutiva aqui apresentada coincide com a atualização irreversível dos rótulos em [4].

símbolo de negação associado ao símbolo de relação I. Dessa forma, podemos expressar a existência de um conjunto conversor de tamanho 2 pela seguinte fórmula de primeira-ordem com o operador de menor ponto-fixo

$$\exists x_1 \exists x_2 \forall y \wedge_{i=1}^{2} X(x_i) \wedge (x_1 \neq x_2) \wedge LFP[\varphi(x, X)_{irr}](y).$$

Assim, o problema da está em P por meio equivalêcia fornecida pelo Teorema de Immerman-Vardi $[1, 7]$ que caracteriza $P =_{ord} FO[\text{LPF}]$. Mais ainda, o problema está em P para qualquer valor de k fixo. Olhando para o problema do conjunto conversor parametrizado pelo tamanho da solução, podemos concluir, imediatamente que o problema está em XP (A classe dos problemas cujas fatias estão em P).

Entretando, a fórmula anterior pode ser utilizada para dizer que o problema pertence classe parametrizada W[2], um resultado de [3] que garante que

$$W[2] = \bigcup_{u \geq 1} \bigcup_{s \geq 1} \text{slicewise-}\Sigma_{2,u}\text{-BOOL}(\text{LFP}^s).$$

Basta considerar $u = 1$ e $s = 2$, onde u representa um limite para o bloco de quantificadores universais e s o posto de quantificadores da fórmula φ_{irr}.

Considerando o problema do conjunto conversor para um processo *reversível*, podemos observar que o conjunto conversor não é estritamente crescente. Assim, não conseguiríamos expressar a atualização do conjunto conversor por meio de uma fórmula positiva no símbolo de relação I. Assim, partindo de um conjunto inicial I, conseguimos definir os próximos vértices infectados usando a fórmula

$$\varphi(x, I)_{ref} := \exists y_1 \exists y_2 (y_1 \neq y_2) \wedge I(y_1) \wedge I(y_2) \wedge E(y_1, x) \wedge E(y_2, x) \wedge \neg I(x).$$

Dessa forma, aplicando o operador de ponto-fixo parcial, podemos construir a seguinte fórmula

$$\exists x_1 \exists x_2 \forall y \wedge_{i=1}^{2} X(x_i) \wedge x_1 \neq x_2 \wedge PFP[\varphi(x, X)_{ref}](y)$$

que nos garante que o problema está em PSPACE. Do ponto de vista parametrizado, a definição apresentada para o problema tem todas as suas fatias em PSPACE. Assim, o problema tem uma classificação imediata em XPSPACE, que é um limite superior muito alto.

Contudo, podemos conjecturar que o limite superior para o caso reversível se encaixa na família de lógicas descritas por

$$\bigcup_{u \geq 1} \bigcup_{s \geq 1} \text{slicewise-}\Sigma_{2,u}\text{-BOOL}(\text{PFP}^s),$$

que dispõe entre as classes parametrizadas candidatas aquelas apresentadas nos trabalhos [8], versões parametrizadas de classes da hierarquia polinomial.

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The Logic of Impossible Truths: A paraconsistent but non-dialetheist framework for paradoxes and other impossibilities

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Abstract

The intuitive reasoning behind the Liar Paradox can be put by two counterfactual sentences: i) If the Liar sentence were true, it would also be false. ii) If the Liar sentence were false, it would also be true. From i and ii, we conclude that the Liar sentence can be consistently evaluated neither as true nor as false. This conclusion can also be seen as showing that the aforementioned counterfactuals have impossible (at least in a logical sense) antecedents.

Counterfactual conditionals with impossible antecedents are usually known as counterpossibles. Counterpossibles have their own puzzles. It is not easy to see, for instance, whether and how a counterpossible would be evaluated as a false sentence. In the standard worldly-based approach provided by Lewis [4] and Stalnaker [6], counterpossibles are vacuously true, since there is no possible world in which the antecedent is true. Some ([2], [3], [5], etc.) have proposed a non-vacuist extension of worldly-based semantics by the insertion of impossible worlds.

Paraconsistency is behind both the intuitions and technical features of non-vacuism. From the perspective of semantics, there seems to be a straight link between Paraconsistency and Dialetheism, the thesis according to which some contradictions are actually true. A paraconsistent logic can be characterized as one in which the Explosion Rule ($\alpha, \neg\alpha \vDash \beta$, for any sentences α and β) doesn't hold without restrictions. Thus, from a paraconsistency perspective, there is at least one sentence α and a case (structure, valuation, interpretation, frame, etc.), such that, both α and $\neg\alpha$ hold in it.

In this talk, I intend to offer a non-dialetheist and non-vacuist paraconsistent framework (the Logic of Impossible Truths, LIT) for both counterpossibles and semantic paradoxes, like the Liar and its more closely related versions. LIT is based on a situation semantics (based on [1]) with a compatibility negation and a *ceteris paribus* conditional. As I'm going to show using this framework, there are no possible (consistent) worlds, but actual situations are all consistent ones. All contradictions belong to impossible non-actual situations but some counterpossibles are actually false. Thus, there might be a new route to think about the intuitive reasoning behind the Liar Paradox, since it is based on two counterpossible sentences.

Keywords. Liar Paradox, Counterpossibles, Situation Semantics, Impossible.

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How to reason with imprecise probability and possibility

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Abstract

Even against the Principle of Explosion or *Ex Contradictione Sequitur Quodlibet* of traditional logic, contradictory information may allow for some reliable inferences. This talk intends to show some developments for dealing with contradictory or missng information and contradictory uncertainty theory represented by probability and possibilistic measures based on the Logics of Formal Inconsistency ([3]).

The tsunami of information received nowadays concentrates two types of uncertainty: one in which reasoning accesses too little information (ignorance, caused by informational gaps), and another in which reasoning accesses too much, usually conflicting information (contradiction, caused by informational gluts).

The working hypothesis underlying this research is that probability theory and possibility theory (an uncertainty theory devoted to the handling of incomplete and also contradictory information, regarded as imprecise probability) can both be regarded as dependent on logic, in the sense of not only generalizing propositional logic, while also generalizing non-standard logics (see [2], [6]).

Possibility theory and probability in non-standard logics show some interesting connections to the notion of evidence. Evidence can be either contradictory or incomplete, or both, thus requiring paracomplete and paraconsistent logics.

A logic with such characteristics, LET_F, extending Belnap-Dunn's logic of first-degree entailment FDE with a classicality operator $\circ A$ that recovers classical logic for formulas in its scope was first proposed in [4]. A probabilistic semantics for LET_F able to quantify the amount of evidence available for a judgement A appears in [6]. The main intuition is that information can be positive (A is true), negative (A is false) missing (no evidence for A) or conflictig (contradictory), while a formula $\circ A$ means that the information about A, either positive or negative, is reliable. This proposal expands the interpretation of FDE and also of Nelson's **N4**, as information-based logics, adding to the four scenarios expressed by such logics two new situations: reliable (conclusive) information towards truth (A is conclusively true), or reliable (conclusive) information towards falsity (A is conclusively false) (cf. [1], [5]). LET_F can also be endowed with possibility measures, opening new perspectives as a tool of reason.

I argue that such topics are of undoubted interest for the philosophy of science, for the foundations of Artificial Intelligence and for reasoning in general.

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Partitions of topological spaces and a club-like principle

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Abstract

The topic of partitions of topological spaces is a nice example of the interaction of Ramsey theory and topology. One could consider the early appearances of this topic to date back to the 70's in the works of [2] and [8] as well as some problems of [3], dealing mainly with ordinals. Later this study would be expanded to properly include topological spaces as in [6] and a later discussion in [7].

When we partition topological spaces, we hope to obtain a homogeneous set which is topologically relevant - for instance, a set homeomorphic to a well known topological space.

Let X be a topological space and α an ordinal. The expression

$$X \to (top\ \alpha)^1_\omega$$

means that, for all $f \in \kappa^\kappa$, there is an $\eta \in \kappa$ and a subset $Y \subseteq f^{-1}[\{\eta\}]$ such that Y is homeomorphic to α with the order topology.

The notation from the usual partition calculus is borrowed for partitions of topological spaces and differently from partition calculus when the superscript is equal to one it does not always signal an easy problem.

Notation: If X is a topological space and $x \in X$, then $\chi(x, X)$ is the least cardinality of a local basis at x, $\chi(X)$ is the least upper bound of all $\chi(x, X)$ with $x \in X$ and $\mathfrak{c}$ denotes $2^{\aleph_0}$, the cardinality of the continuum.

In joint work with Carvalho and Junqueira in [1] we fixed a gap in the original proof of the following theorem:

Theorem A [5, Theorem 1] Let X be a regular topological space such that $X \to (top\ \omega+1)^1_\omega$ and $\chi(X) < \mathfrak{b}$. Then $X \to (top\ \alpha)^1_\omega$ for all $\alpha < \omega_1$.

In contrast with Theorem 1, Komjath and Weiss proved that it is consistent with ZFC that $\chi(X) < \mathfrak{b}$ is the best possible bound in Theorem A. For that they consider the combinatorial principle $\diamondsuit_{\omega_1}$, which states that there exists a sequence $\langle A_\alpha \mid \alpha \in acc(\omega_1)\rangle$ such that for every $X \subseteq \omega_1$ the set $\{\alpha \in acc(\omega_1) \mid X \cap \alpha = A_\alpha\}$ is stationary. Using $\diamondsuit_{\omega_1}$, in [5, Theorem 4], they constructed a topological space (X, τ) with the following features: $\chi(X) = \mathfrak{b}$, $(X, \tau) \to (\omega+1)^1_\omega$, and $(X, \tau) \not\to (\omega^2 + 1)^1_\omega$.

In [5] it is observed that if $(X, \tau) \to (\omega + 1)^1_\omega$, then $(X, \tau) \to (\omega^2)^1_\omega$. Therefore the example constructed using $\diamondsuit_{\omega_1}$ is optimal.

Notice that if $\langle A_\alpha \mid \alpha \in acc(\omega_1)\rangle$ witnesses $\diamondsuit_{\omega_1}$, then all $x \subseteq \omega \subseteq \omega_1$ appears in $\langle A_\alpha \mid \alpha < acc(\omega_1)\rangle$ and therefore $\mathfrak{c} = \aleph_1$.

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It remained open whether there exists a topological space (X,τ) such that $(X,\tau) \to (\omega+1)^1_1$, $(X,\tau) \not\to (\omega^2+1)$ together with $\omega_1 = |X| < \mathfrak{c}$ or $\chi(X) < \mathfrak{c}$. In [1] we proved that actually both conditions can hold simultaneously:

Theorem B [1, Theorem 4.3] It is consistent relatively to ZFC that there exists a topological space (X,τ) such that

- $|X| = \chi(X) = \mathfrak{b} = \omega_1 < \mathfrak{c}$,
- $(X,\tau) \to (\omega_1)^1_\omega$
- $(X,\tau) \not\to (\omega^2+1)^1_\omega$

For that we define a new combinatorial principle $\clubsuit_F$ which captures the main usage of $\diamondsuit_{\omega_1}$ in the construction due to Komjath and Weiss. The principle $\clubsuit_F$ states that there exists a sequence $\langle A^m_\alpha \mid \alpha \in acc(\omega_1) \wedge m \in \omega\rangle$ such that for every $\alpha \in acc(\omega_1)$ we have $sup(A_\alpha) = \alpha$ and for every $f : \omega_1 \to \omega$ there are $\alpha < \omega_1$, $m \in \omega$, $n \in \omega$ such that $A^m_\alpha \subseteq f^{-1}[\{n\}]$ and $f(\alpha) = n$.

We prove that $\clubsuit_F$ is a consequence of $\diamondsuit^*_{\omega_1}$ and use a CS*-iteration of Cohen reals from [4] to prove the consistency of $\clubsuit_F$ with $\neg CH$. Then we use $\clubsuit_F$ to prove that in the forcing extension there is a topological space (X,τ) as we seek.

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On some extensions of da Costa logic

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Abstract

In 2009 [3] and 2011 [4], Graham Priest introduced propositional and first-order da Costa logic, respectively. Propositional da Costa logic may be defined as positive propositional logic expanded with a unary connective D that may be seen as a negation satisfying *tertium non datur* and *modus tollendo ponens* for derivable disjunctions. As a result, intuitionistic negation is definable. It is easy to see that D behaves as a paraconsistent negation. In 2013 [1], paying attention to an observation due to Urbas (see Example E1 in [5, p. 345]), we proved that propositional da Costa logic is *strictly* paraconsistent.

On this ocassion, we consider the different extensions of propositional da Costa logic that result by the addition of schemas $\neg\alpha \vee \neg\neg\alpha$, its dual $DDx \to \neg Dx$, and, algebraically motivated, what we call *weak regularity*, that is, the axiom schema $(\alpha \wedge Dx) \to (\beta \vee \neg\beta)$, where D is to be understood as said above and the other symbols for connectives may be understood as usual.

In particular, we pay attention to the fact that in some cases both the conditional and its dual become definable. Moreover, not only D behaves as a paraconsistent connective as there is a definable connective for which we use the notation $\sim$ that is also paraconsistent.

We prove that in certain case $\sim$ behaves as an involution, a notion considered in this context a long time ago (see, for example, [2]). Finally, as a necessity operator $\Box$ is also available, we also pay attention to the different schemas involving $\Box$ that hold in the mentioned extensions.

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On the logical relationship between the Peano and Lawvere axioms for the sequence of natural numbers

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Abstract

We study the logical relationship between two sets of axioms for the sequence of natural numbers. The first one is the very well known set of three Peano Axioms. The second one is a set of two axioms adapted from the axiom proposed by F. W. Lawvere in [PNAS, 52:1506–1511,1964] as an axiomatization of the sequence of natural numbers.

A *model* is a system consisting of a set N, an element 0 of N, and a unary operation S on N. A model $\langle N, 0, S\rangle$ is a *Peano model*, or simply a *P-model* if it satisfies the following three statements:

Zer. For all $x \in N$, $S(x) \neq 0$.

Inj. For all $x, y \in N$, if $S(x) = S(y)$, then $x = y$.

Ind. For all $X \subseteq N$, if $0 \in X$ and $S(x) \in X$ whenever $x \in X$, then $N \subseteq X$.

Axioms Zer, Inj, and Ind are as proposed by G. Peano in [7]. Axiom Zer—the *Zero Axiom*—states that 0 is not in the range of the S operation. Axiom Inj—the *Injectivity Axiom*—states that the S operation is injective. Axiom Ind—the *Induction Axiom*—states that N is the only subset of N that both contains 0 as an element and is closed under the S operation.

A model $\langle N, 0, S\rangle$ is a *Lawvere model*, or simply a *L-model* if it satisfies the following two statements:

Exi. For all set X, element x of X, and unary operation f on X, there exists a mapping $h : N \to X$ such that $h(0) = x$, and $h(S(y)) = f(h(y))$ whenever $y \in N$.

Ndu. For all set X, element x of X, unary operation f on X, and mappings $h_1, h_2 : N \to X$, if $h_1(0) = h_2(0) = x$, and for every $y \in N$, $h_1(S(y)) = f(h_1(y))$ and $h_2(S(y)) = f(h_2(y))$, then $h_1 = h_2$.

Axioms Exi and Ndu are adapted from the one considered by F. W. Lawvere in [2,3]. Axiom Exi—the *Existence Axiom*—states that for any model $\langle X, x, f\rangle$ there exists at least one homomorphism h from $\langle N, 0, S\rangle$ into $\langle X, x, f\rangle$. Axiom Ndu— the *Non-duplicity Axiom*—states that for any model $\langle X, x, f\rangle$ there exists at most one homomorphism h from $\langle N, 0, S\rangle$ into $\langle X, x, f\rangle$.

R. Dedekind [1] proved, essencially, that every P-model is a L-model. F. W. Lawvere [2,3] proved that, reciprocally, every L-model is a P-model. In this note, we refine these results, proving the results below. To state the issues more clearly, we write "$\Phi \vdash \varphi$" instead of "there is a proof of φ from Φ in the second order system described in [8]," where Φ is a set of statements and φ is a statement in the referred second order language. As usual, we write "$\Phi \nvdash \varphi$" instead of "not $\Phi \vdash \varphi$".

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1. $\{\mathsf{Zer}, \mathsf{Inj}, \mathsf{Ind}\} \vdash \mathsf{Exi}$. Moreover, for any proper non-empty subset Σ of $\{\mathsf{Zer}, \mathsf{Inj}, \mathsf{Ind}\}$, we have $\Sigma \nvdash \mathsf{Exi}$.
2. $\{\mathsf{Ind}\} \vdash \mathsf{Ndu}$.
3. $\{\mathsf{Exi}\} \vdash \mathsf{Zer}$.
4. $\{\mathsf{Exi}, \mathsf{Ndu}\} \vdash \mathsf{Inj}$. Moreover, $\{\mathsf{Exi}\} \nvdash \mathsf{Inj}$ and $\{\mathsf{Ndu}\} \nvdash \mathsf{Inj}$.
5. $\{\mathsf{Ndu}\} \vdash \mathsf{Ind}$.

It is worth to point out that:

(1) To state and "prove" $\{\mathsf{Ind}\} \vdash \mathsf{Exi}$ is a common mistake (see e.g. [5]).

(2) In comparing $\{\mathsf{Zer}, \mathsf{Inj}, \mathsf{Ind}\}$ with $\{\mathsf{Exi}, \mathsf{Ndu}\}$ as axioms, the widespread proof is that $\{\mathsf{Exi}, \mathsf{Ndu}\} \vdash \mathsf{Ind}$ (see e.g. [4]), but results 2 and 5 above show that, more perspicuously, Ind and Ndu are second order equivalent.

(3) There is an area of investigation in which the results presented above can be adapted and extended, notably the one related to the so-called General Principles of Induction and Recursion, by R. Montague [6]. We leave this possibility for future work.

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O estruturalismo lógico de Karl Popper

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Resumo

A presente comunicação tem por objetivo apresentar os ainda pouco conhecidos trabalhos de Karl Popper acerca da lógica dedutiva, escritos entre 1947 e 1949 [7–12]. Nesses artigos, Popper afirma oferecer uma generalização da definição de consequência lógica de Tarski [16]. Por simplicidade, nos concentraremos em dois artigos de 1947 [7, 8], elaborando apenas a parte de seu argumento que diz respeito à lógica proposicional. A teoria apresentada por Popper, recebida sobretudo como uma proposta de fundamentação da lógica, foi alvo de duras críticas (e.g. por Kleene [4] e McKinsey [6]; uma exposição e bibliografia mais completas acerca da proposta de Popper e suas críticas pode ser encontrada em [1] e [14]). Essas críticas levaram o filósofo a abandonar seu projeto e a posteriormente afirmá-lo como um erro. Isso se deveu, sobretudo, ao fato de Popper ter sido incapaz de tornar suas intenções suficientemente claras e de muitas de suas ideias estarem radicalmente distantes do consenso acerca da lógica na época, levando seus críticos, ao identificarem erros técnicos de fato existentes, a expressar dúvidas quanto à viabilidade de seu projeto. No entanto, é possível hoje reconstruir a teoria de Popper, à luz dos desenvolvimentos posteriores da lógica, de maneira a torná-la coerente e frutífera, permitindo ainda que identifiquemos em seu trabalho a antecipação de temas que se tornariam comuns na literatura posterior. Em particular, é notável a identificação por parte de Popper das propriedades estruturais da relação de dedutibilidade, as quais já haviam sido estudadas por Gentzen [2], porém seu trabalho era ainda pouco conhecido. Mais recentemente, Schroeder-Heister [14] defendeu a ideia de que a teoria de Popper poderia ser entendida como uma tentativa de delimitação das constantes lógicas das não-lógicas, ou seja, que Popper teria sido capaz de apresentar um critério válido de logicalidade. Posteriormente, em [15], Schroeder-Heister apresenta a teoria de Popper de maneira mais geral, como uma abordagem estruturalista acerca da lógica, aproximando-a e relacionando-a com o trabalho de Koslow [5]. Nessa leitura, o objetivo de Popper é entendido como o de apresentar a lógica como uma teoria metalinguística acerca de relações de consequência ou dedutibilidade, em termos das quais seria possível caracterizar as operações lógicas. Longe, portanto, de se enquadrar como uma proposta fundacionista, que contradiria inclusive o anti-fundacionismo característico da filosofia de Popper, esses artigos buscariam caracterizar a lógica de uma maneira neutra, como uma teoria metalinguística descritiva acerca de diferentes 'estruturas de dedução'. Uma tal estrutura de dedução pode ser definida como um par $\langle D, \vdash \rangle$, onde D é um conjunto e $\vdash$ é uma relação entre os elementos de D respeitando, pelo menos, os seguintes princípios de 'reflexividade' e 'transitividade generalizada', respectivamente, os quais Popper chama de 'regras primitivas':

$$\Gamma, A \vdash A$$
$$(\Gamma \vdash B_1 \;\&\; \dots \;\&\; \Gamma \vdash B_n) \Rightarrow (B_1, \dots, B_n \vdash C \Rightarrow \Gamma \vdash C)$$

onde Γ é um subconjunto de D, $A, B, C, \dots$ são elementos de D, e $\&, \Rightarrow$ correspondem, respectivamente, à conjunção e à implicação metalinguísticas. Esses princípios são chamados por Popper de 'absolutamente válidos', pois não fazem referência a conectivos lógicos de alguma linguagem-objeto particular. Assim, as operações lógicas disponíveis em uma estrutura podem ser caracterizadas metalinguisticamente por meio de seu comportamento inferencial, independentemente da existência sintática de conectivos que as representem. Por exemplo, um elemento A de D é dito uma conjunção de A_1 e A_2, a menos de interdedutibilidade, se e somente se, para todo $B \in D$, $A \vdash B$ se e somente se $A_1, A_2 \vdash B$. Uma tal caracterização, que visa, segundo

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Popper, estabelecer a 'força lógica' da conjunção, é chamada por ele de uma *definição inferencial*. De maneira semelhante, todas as outras operações lógicas podem ser caracterizadas. O fato de uma operação lógica ser definida a menos de interdedutibilidade em uma estrutura é explorado por Popper como um critério de logicalidade, o qual ele demonstra que falha para a negação minimal de Johansson [3], assim como para certos conectivos irregulares, antecipando a discussão sobre conectivos como o *tonk* de Prior [13]. Contrariamente a esse, no entanto, podemos dizer que Popper antecipa uma filosofia inferencialista acerca da lógica, que teve importantes desenvolvimentos nas décadas seguintes, além de se aproximar, paralelamente a Gentzen, dos desenvolvimentos posteriores da semântica prova-teorética, devidos sobretudo a Prawitz. Uma profusão de outros temas de interesse tanto para a lógica formal quanto para a filosofia da lógica pode ser encontrada nesses trabalhos de Popper. Por essa exposição das características gerais da teoria da dedução apresentada por Popper, buscando situá-la tanto em seu contexto imediato como em sua recepção posterior, pretendemos contribuir com a divulgação de um importante, porém pouco conhecido, episódio no desenvolvimento da lógica contemporânea, protagonizado por um dos mais importantes filósofos e epistemólogos do século XX.

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Paradefinite Ivlev-like modal logics based on FDE

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Abstract

In a series of works started in the 1970's (see, for instance, [11], [12], [13]), Ju. Ivlev proposes a novel approach to non-normal modal logics (that is, without validating the *necessitation* rule NEC) in which $\Box$ is not congruential, and the semantics is not given by neighborhood semantics, but it is presented by finite non-deterministic matrices. This approach anticipated the use of non-deterministic matrices (or Nmatrices) for non-classical logics proposed by A. Avron and I. Lev (see [1], [2]). Ivlev's approach was revisited and extended in [7] and [17]. In particular, in [7] the five systems Tm, $T4m$, $T45m$, TBm, $T4Bm$ were presented, each one characterized by a single 4-valued Nmatrix, corresponding, respectively, to weaker versions of modal systems **T**, **T4**, **T45**, **TB** and **T4B**. In [8] this semantics was extended to a class of more general Nmatrices called *swap structures*, in which each truth-value is a triple $z = (z_1, z_2, z_3)$ (called *snapshot*) such that each coordinate z_i (with values in a given Boolean algebra $\mathcal{A}$) represents a truth-value for the formulas φ, $\Box\varphi$ and $\Box{\sim}\varphi$, respectively (where $\sim$ is the classical negation). Swap structures were proposed in [5, Chapter 6] as a general non-deterministic framework to deal with paraconsistent and other non-classical logics. Swap structures are a non-deterministic generalization of the twist-structures introduced independently by M. Fidel [10] and D. Vakarelov [19], in which the snapshots are pairs and obey to an algebraic (deterministic) structure (see below). In [8] it was shown that each original 4-valued Nmatrix proposed in [7] is recovered by taking snapshots over the 2-element Boolean algebra with domain $\{0, 1\}$. The four truth-values are, therefore, $T = (1, 1, 0)$ (meaning that φ is necessarily true), $t = (1, 0, 0)$ (φ is contingently true), $f = (0, 0, 0)$ (φ is contingently false) and $F = (0, 0, 1)$ (φ is necessarily false). Obviously the designated values are T and t.

Also in the 1970's, N. Belnap and J.M. Dunn introduce in [3], [4], [9] the first-degree entailment logic (FDE), a 4-valued logic nowadays known as Belnap-Dunn logic. This logics intends to formalize the way a computer deals with information. Indeed, there are basically four possibilities when a computer receives information from different sources: an item come in as asserted ('True'), as denied ('False'), both possibilities ('Both') or none of them ('None') (see [3]). These four possibilities correspond to the four values of the logic FDE. It is convenient identifying these values with ordered pairs (a, b) of 0-1's, obtaining respectively the pairs $(1, 0)$, $(0, 1)$, $(1, 1)$ and $(0, 0)$. These pairs, in which the first and second coordinate correspond, respectively, to asserting and denying a given item φ of information (symbolized respectively by φ and $\neg\varphi$), constitute a twist-structure for FDE, in which the logical operators (conjunction, disjunction and negation are defined in a natural way: $(a, b) \wedge (c, d) = (a \sqcap c, b \sqcup d)$; $(a, b) \vee (c, d) = (a \sqcup c, b \sqcap d)$; and $\neg(a, b) = (b, a)$ (here, $\sqcap$ and $\sqcup$ are the infimum and supremum in the 2-element Boolean algebra $\{0, 1\}$). These operations validate the De Morgan laws, and the negation is idempotent: $\neg\neg x = x$, defining so a (bounded) De Morgan lattice (observe, however, that neither $\top$ nor $\bot$ are definable in FDE). A. Pynko [18] studied several expansions of FDE from the proof-theoretic and algebraic point of view, by using twist structures, introducing an interesting implication over FDE which defines a logical system called $\mathcal{IDM}4$ (see [18, p. 70]). This study was retaken by S. Odintsov [14] in the context of Nelson's logic $N4$ and its algebraic semantics. In Section 5 of such article, Odinstov recovers $\mathcal{IDM}4$ under the name of $B_4^{\rightarrow}$, as an special model of $N4^{\bot}$, the expansion of $N4$ by a bottom $\bot$ such that $\neg\bot$ is a theorem. As observed by Odintsov, the implication $\rightarrow$ can be defined, in terms of twist structures over $\{0, 1\}$, as $(a, b) \rightarrow (c, d) = (a \Rightarrow c, a \sqcap d)$

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(where $\Rightarrow$ is the Boolean implication). The second component of the implication in the twist structure reflects the fact that, in $N4$, $\neg(\varphi \to \psi)$ is equivalent to $\varphi \wedge \neg\psi$.

In [6] W. Carnielli and A. Rodrigues propose a novel and fruitful interpretation of $N4$ in terms of *evidence*, under the name of BLE (*Basic Logic of Evidence*). Hence, the first and second components of a pair (a, b) can be seen as evidence *in favor* of φ and $\neg\varphi$, respectively, being this equivalent to evidence in favor of φ and *against* φ, respectively, or even evidence against $\neg\varphi$ and φ, respectively. They consider a family of *logics of evidence and truth* (LETs) based on FDE, $N4$, $B_4^{\to}$ and other extensions of FDE, by adding a *classicality* unary connective $\circ$ which allows to locally recover the validity of the explosion law and the excluded middle, in a similar way as the explosion law is locally recovered in the *logics of formal inconsistency* (**LFI**s, see for instance [5]). Namely, in LETs $\circ\varphi$ derives $\varphi \vee \neg\varphi$, and everything follows from $\varphi \wedge \varphi \wedge \circ\varphi$. Observe that, by definition, all these systems are both paraconsistent and paracomplete w.r.t. $\neg$. Such kind of logics are called *paradefinite*, or *paranormal*, or *non-alethic*.

The objective of the present proposal is to combine the 4-valued Ivev-like modal systems studied in [7] with $B_4^{\to}$ (considered as a logic of information). From the semantical perspective, the idea is to combine the modal 4-valued swap structures (Nmatrices) introduced in [8] with the twist structures (matrices) for $B_4^{\to}$. This requires considering 4-dimensional snapshots $z = (z_1, z_2, z_3, z_4)$ such that each coordinate z_i (with values 0 or 1) represents a possible truth-value for the formulas φ, $\Box\varphi$, $\Box{\sim}\varphi$ and $\neg\varphi$, respectively, where $\sim$ is the classical negation and $\neg$ is the (paraconsistent and paracomplete) negation of $B_4^{\to}$. That is, we enrich the original 3-dimensional snapshots T, t, f and F mentioned above with a fourth coordinate, of an epistemic character. This would produce, in principle, eight snapshots, given that each original snapshot would split into two possibilities, according to the informational status of φ (evidence in favor or against φ). However, in case φ is necessarily true, which corresponds to $T = (1, 1, 0)$, there could be no evidence against φ, and so the fourth coordinate (representing $\neg\varphi$) can only be 0. Analogously, if φ is necessarily false, corresponding to $F = (0, 0, 1)$, there could be no evidence in favor of φ, and so the fourth coordinate (representing $\neg\varphi$) can only be 1. This produces a universe of six snapshots, with only three of them being designated (the ones with first coordinate 1). With suitable definitions of the (non-deterministic) swap operations, it is a obtained a 6-valued characteristic Nmatrix for each one of the five Ivlev-like modal logics mentioned above. This produces five different paradefinite Ivlev-like modal logics combined with $B_4^{\to}$, that is, extending FDE. The formula $\circ\varphi = (\varphi \vee \neg\varphi) \wedge {\sim}(\varphi \wedge \neg\varphi)$ defines a classicality operator for these logics in the sense of LETs mentioned above. Finally, a sound and complete Hilbert-style axiomatization is found for the five systems, by gathering: (1) the axiomatization of $N4^{\perp}$ given in [14] plus the Peirce law $\varphi \vee (\varphi \to \psi)$ (which axiomatizes $B_4^{\to}$); (2) the original axiomatization of the corresponding Ivlev-like modal logic given in [8]; and (3) some bridge principles involving $\Box$, $\sim$ and $\neg$ establishing, for instance, the equivalence between $\Box\neg\varphi$ and $\Box{\sim}\varphi$, as well as the equivalence between $\Box\varphi$, $\Box\neg\neg\varphi$ and $\Box\neg{\sim}\varphi$ (the equivalence between $\Box\varphi$ and $\Box{\sim}{\sim}\varphi$ already holds in Tm). Clearly, each of these systems is decidable by its characteristic 6-valued Nmatrix.

As related works we can mention the (normal) modal systems studied by S. Odinstov and H. Wansing in [15] and [16], by combining the minimal normal modal logic **K** with $B_4^{\to}$. As in our case, and in order to harmonize both logics, they consider some bridge principles between $\Box$, $\sim$ and $\neg$, some of them stronger than ours. They propose twist structures semantics defined over Boolean algebras with modal operators, a.k.a. *modal algebras*, given the validity of NEC.

The extension of our framework to Ivlev-like deontic systems seems to be perfectly feasible, by adapting the swap structure semantics for such systems introduced in [8]. We consider that our approach is an interesting alternative to the standard modal logics, since it avoids several problems associated to the unrestricted use of the necessitation rule, on the one hand, and they are decidable by finite-valued Nmatrices. Besides the potential applications to informational systems, because of its paraconsistent and paracomplete character, the expressivity of the proposed logics suggests applications to the analysis of modal paradoxes, among other domains.

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From inconsistency to incompatibility

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Abstract

When studying paraconsistent logics, specially logics of formal inconsistency (**LFI**$'s$), it is common to soften the explosion law $\alpha, \neg\alpha \vdash \beta$ by addition of the well-behavior, or consistency, of the formula α, thus obtaining $\circ\alpha, \alpha, \neg\alpha \vdash \beta$, where $\circ\alpha$ means α is consistent. We can interpret $\circ\alpha$ as stating that α can not coexist with $\neg\alpha$, that the two are incompatible.

So, we attempt to generalize inconsistency to incompatibility: by considering the signature with connectives $\vee$, $\wedge$, $\rightarrow$ and $\uparrow$, the last one a binary connective for which $\alpha \uparrow \beta$ is understood as "α is incompatible with β", we define a first, simplest system of incompatibility **bI** by taking the axioms and rules of inference of the positive fragment of classical propositional logic (**CPL**), plus the axioms

$$\alpha \vee (\alpha \rightarrow \beta), \qquad (\alpha \uparrow \beta) \rightarrow (\beta \uparrow \alpha) \qquad \text{and} \qquad (\alpha \uparrow \beta) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma)).$$

It is clear that $(\alpha \uparrow \beta) \rightarrow (\beta \uparrow \alpha)$ stands for the commutativity of incompatibility, implying that if α is incompatible with β, β is incompatible with α and vice-versa; curiously, while **bI** is not characterizable by finite Nmatrices, the logic **bI**$^-$, equal to **bI** except for not having the commutativity of incompatibility, can be characterized by a two-valued Nmatrix. Meanwhile, the axiom $(\alpha \uparrow \beta) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$ is a modified explosion law, signifying that if two formulas are incompatible, having both of them to simultaneously hold trivializes an argument.

We provide semantics for **bI** through bivaluations and Fidel structures, *i.e.* Boolean algebras equipped with non-deterministic operators. Both of these happen to be instances of restricted non-deterministic matrices, also known as RNmatrices, non-deterministic algebraic semantics of the form $\mathcal{M} = (\mathcal{A}, D, \mathcal{F})$ for $\mathcal{A}$ a multialgebra on a given signature Σ, D a subset of its universe and $\mathcal{F}$ a set of homomorphisms $\nu : \mathbf{F}(\Sigma, \mathcal{V}) \rightarrow \mathcal{A}$,[1] giving us a consequence operator such that

$$\Gamma \vDash_{\mathcal{M}} \varphi \quad \text{iff} \quad \nu[\Gamma] \subseteq D \quad \text{implies} \quad \nu(\varphi) \in D.$$

Furthermore, the two-valued Fidel structure for **bI** leads to decision methods for this logic both by a tableaux calculus and row-branching, row-eliminating truth tables.

Attempting to extend **bI**, it is easy to find unsuitable axioms which collapse **bI** back to propositional logic, with $\uparrow$ becoming the Sheffer's stroke; but we can obtain the logics **nbI**, and **nbIciw**, **nbIci** and **nbIcl** by addition of the axioms

$$\alpha \vee \neg\alpha, \quad \text{and} \quad (\alpha \uparrow \neg\alpha) \vee (\alpha \wedge \neg\alpha), \quad \neg(\alpha \uparrow \neg\alpha) \rightarrow (\alpha \wedge \neg\alpha) \quad \text{and} \quad \neg(\alpha \wedge \neg\alpha) \rightarrow (\alpha \uparrow \neg\alpha)$$

to, respectively, **bI** and **nbI**. By translating a formula $\circ\alpha$ of **LFI**$'s$ as $\alpha \uparrow \neg\alpha$, we obtain conservative translations of the logics **mbC**, **mbCciw**, **mbCci** and **mbCcl** into, respectively, **nbI**, **nbIciw**, **nbIci** and **nbIcl**, showing that incompatibility truly generalizes inconsistency in a non-trivial way.

Finally, we show that our logics of incompatibility, beyond being not algebraizable according to Blok and Pigozzi, also can not be characterized by finite Nmatrices or Rmatrices: of course, this means that it is surprisingly difficult to provide semantics for these logics.

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[1]By $\mathbf{F}(\Sigma, \mathcal{V})$ we mean the algebra of formulas in the signature Σ with propositional variables in $\mathcal{V}$.

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A Graph Logical Framework

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Abstract

We conjecture that the relative unpopularity of logical frameworks among practitioners is partly due to their complex meta-languages, which often demand both programming skills and theoretical knowledge of the meta-language in question for them to be fruitfully used. We present ongoing work on a logical framework with a meta-language based on graphs. A simpler meta-language leads to a shallower embedding of the object language, but hopefully leads to easier implementation and usage. A graph-based logical framework also opens up interesting possibilities in time and space performance by using heavily optimized graph databases as backends and by proof compression algorithms geared towards graphs. Deductive systems can be specified using simple domain-specific languages built on top of the graph database's query language. There is support for interactive (through a web-based interface) and semiautomatic (following user-defined tactics specified by a domain-specific language) proof modes. Proofs can be exported to print-friendly (LaTeX) and portable (JSON) formats (and the latter can be imported back). We have so far implemented 17 systems for propositional logic, including Fitch-style, backwards-directed Natural Deduction, and Tableaux systems for intuitionistic and classical logic (with the classical systems reusing the rules of the intuitionistic ones), and Hilbert-style systems for several modal logics.

Deductive systems are regularly proposed without an accompanying implementation. After a deductive system is developed, sometimes an implementation is made, almost as an afterthought. We believe, however, that having such implementations available during the development of such a system is valuable, and even after the system has been fully developed, an implementation is still helpful in understanding it — not to mention in applying it. We postulate that this problem is pervasive because the cost of developing a custom deductive system (or adapting an existing one) is high.

To reduce this implementation cost, we propose a logical framework which uses graphs as its meta-language (following [5]). Once we have graphs as a meta-language, deductive rules become graph transformations, and proofs are represented as graphs.

Other logical frameworks like Isabelle/HOL or Coq have meta-languages that are often more complex than the object languages they are working with. We invert this relationship, by asking users to implement their deductive systems in a simpler language – that of graphs, which are elementary mathematical objects that have been studied for centuries.[1] By choosing as a meta-language a theory that users are likely to already be familiar with, we are able to dramatically reduce the entry barrier for new users.

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[1]Compare that to type theory, which started being developed in the early twentieth century, with much of its influence in logical frameworks deriving from Martin Löf's work in the seventies.

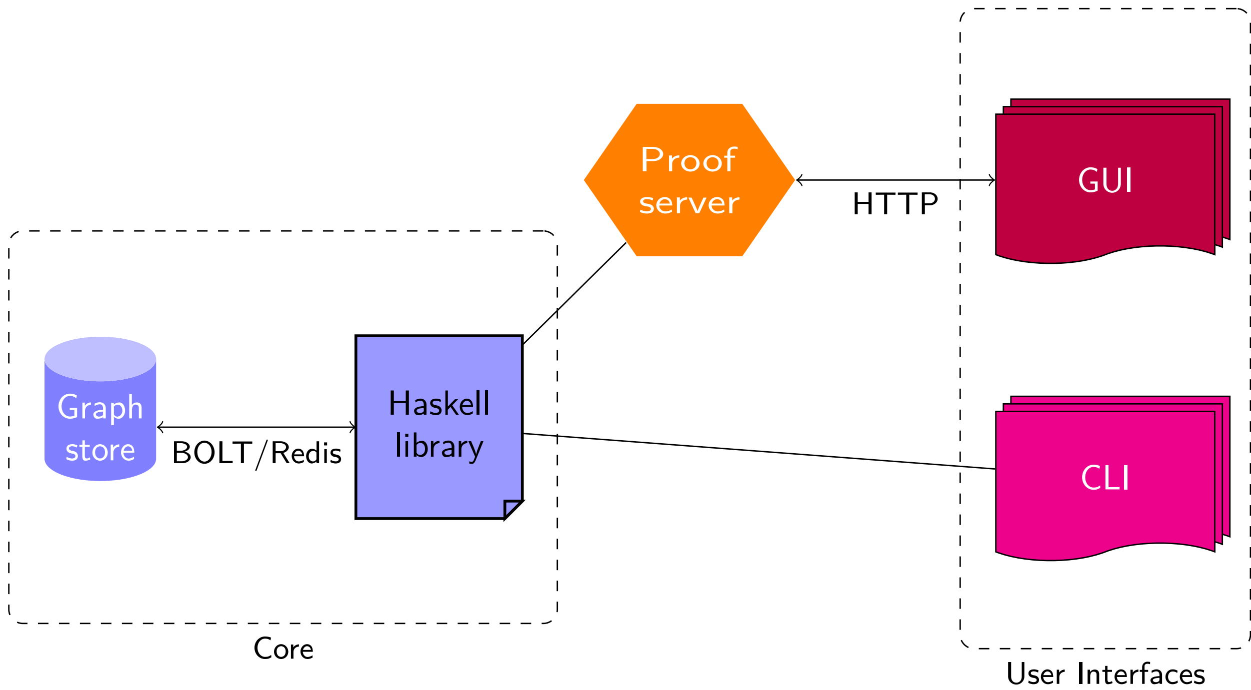

Figure 1: Architecture overview of the graph logical framework

Our second front on a creating a simpler-to-use logical framework is to provide user interfaces that cater to non-programmers. We do still provide programmatic interfaces to our framework, of course, and also offer semi-automatic capabilities with the definition of proof strategies. We strive to have deductive systems implemented in GLF be close to their pen-and-paper versions, however, even at the cost of more difficult formal verification of their correctness.

The logical framework we describe — which we not so creatively call GLF, for Graph Logical Framework — is built as a Haskell library implementing the rule application engine, supported by a graph database back-end. A choice of graph databases is available, provided they implement either of the open standard protocols Bolt or Redis, and employ the Cypher query language. An HTTP proof server and a command-line interface are built on top of the library, and a graphical user interface (in the form of a web application) depends on the proof server to interact with the underlying graph database. An overview of the system architecture of GLF can be seen in Figure Figure 1. An instance of GLF is publicly available at `https://glf.tecmf.inf.puc-rio.br/` (users must register to be able to save proofs, but may try out the system without registration).

Related Work Although many logical frameworks have been developed, few have used graphs as a meta-language. *The incredible proof machine* [2] is a graph-based generic theorem prover. It supports several logics, which can be specified using a simple text format. Although flexible, its design does not seem able to support sub-structural logics and maybe other more sophisticated deductive systems. It also does not offer the ability to customize its interface based on the logic being used, nor does it offer support for semiautomatic theorem proving (in the form of tactics or otherwise). It is meant as an educational tool, not as a formalization tool, and is not supposed to have good performance properties or handle large proofs (we have not ascertained whether it does or not, however).

Porgy (presented in [3], among other works) is a graph rewriting system used to implement a visual modeling system and is not meant to be a logical framework, although it can conceivably be used as such. We could not personally test this system since the latest released version systematically crashed in our machine when following its tutorial. Porgy has a tactic specification language and thus supports semiautomatic use, but as far as we have read, it does not seem to support its use as a library and is thus confined to interactive use. It does not seem to support the programmatic specification of deductive systems, relying on its graphical interface for this task.

The *Cathedral Theorem Proving Platform* [5] is meant to be used as a logical framework from the start, complete with a language for specifying tactics. It is the first (to the best of our knowledge) graph-based logical framework but has not seen wide use since then, nor is the source code available (that we know of).

Another system that is closely related to our work proposal is ELAN [1]. It is a logical framework system implementation based on a many-sorted rewriting logic. Our proposal can be seen as a logical framework based on a graph rewriting system. Using graphs instead of an abstract rewriting system affords us the use of already implemented graph algorithms and makes the system easier to use and understand (since graphs are commonplace and intuitive).

With respect to the user interface, Carnap [4] seems to be the closest work to ours. It too uses a web-based interface, and tries to keep proofs close to pen-and-paper ones. It is focused solely on educational use, however, not providing programmatic use. Its proof interfaces also seem limited to versions of Natural Deduction in Fitch style (or similar styles), while GLF supports both forward and backwards proofs for Natural Deduction, and Tableaux proofs.

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A Ontologia Científica de Quine: questões lógicas e filosóficas

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Resumo

A palavra "ontologia" possui uma longa tradição na filosofia significando, na maioria das vezes a parte da metafísica que trata das estruturas mais gerais daquilo que há. Neste presente resumo busca-se apresentar os principais aspectos da Ontologia desenvolvida pelo filósofo Willard Van Orman Quine, notadamente aqueles relacionados a lógica. Quine está inserido na tradição da filosofia analítica e sua ontologia é um trabalho que ele supõe que deve ser feito de dentro da ciência. Este é texto elaborado a partir da tese em desenvolvimento pelo autor junto ao programa de pós graduação em filosofia da Universidade Federal de Minas Gerais.

O sistema filosófico de Quine tem como característica fundamental uma combinação de naturalismo, empirismo e holismo. O naturalismo vai nos dizer que a ciência é o nosso melhor e mais completo projeto de um sistema empiricamente fundamentado assim como o árbitro final da verdade e da existência. Já o empirismo sustenta que a evidência final para nossas crenças é a evidência sensorial, cabendo à epistemologia explicar o processo de transformação dos inputs que os sentidos recebem, em um farto discurso constituído por nossas teorias cientificas. O holismo, por sua vez, prioriza uma construção integral do conhecimento científico, estabelecendo um sistema de crença em vez de elementos individuais.

No sistema quiniano a evidência da existência de objetos deve ser indireta e extraída de nosso sistema de crenças. Como dito anteriormente, o árbitro final de tais questões é o sistema composto por nossas teorias científicas. Um primeiro passo para esclarecer esse ponto é estabelecer um critério de assunção de compromisso ontológico por uma teoria. Quine fez isso da seguinte maneira:

> When we want to check on existence, bodies have it over other objects on the score of their perceptibility. But we have moved now to the question of checking not on existence, but on imputations of existence: on what a theory says exists. ? To show that a theory assumes a given object, or objects of a given class, we have to show that the theory would be false if that object did not exist, or if that class were empty; hence that the theory requires that object, or members of that class, in order to be true [2]

Em particular, Quine argumenta que a estrutura da teoria regimentada é lógica de primeira ordem com identidade (e, portanto, essa teoria é extensional); e que as variáveis dessa teoria podem abranger apenas conjuntos. Assim como a ciência possui o mesmo objetivo que o conhecimento comum, mas alcança resultados melhores ao descrever a realidade, a linguagem regimentada possui a mesma empreitada que a linguagem comum, sua adoção maximiza as virtudes científicas já parcialmente presentes no discurso comum: estrutura lógica, referência, a facilidade de acesso a métodos algorítmicos. Portanto, a regimentação da nossa teoria cientifica sobre o mundo e o critério de compromisso ontológico permitem esclarecer o que a ciência diz que existe.

Um aspecto importante a destacar aqui é que, no que tange às questões ontológicas e nossas teorias cientificas, a ideia de que a conexão entre a evidência e o modo como falamos dos objetos (em um sistema definido de objetos) não é rígida. Isso significa que podemos reinterpretar um discurso sobre certos objetos em um outro sistema de objetos, desde de que se mantenha fixa a evidência do sistema original. Deste modo, temos a relatividade ontológica.

No ensaio *Things and Their Place in Theories* , Quine escreve: "Structure is what matters to a theory, and not the choice of its objects" [5]. Isso parece suscitar algum conflito com o

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critério quiniano de compromisso ontológico: se os objetos não são o que importa para uma teoria, mas sim a estrutura dessa teoria, com que objetos uma teoria regimentada em uma linguagem de primeira ordem se compromete ontologicamente? Sendo a teoria regimentada, são os objetos que devemos considerar no domínio de uma interpretação da linguagem de uma teoria quando tratamos as questões ontológicas associadas às teorias, ou há algo mais? O que Quine está querendo nos dizer com Estrutura (de uma teoria)? As questões ontológicas e formais para Quine estão imbricadas de um modo bastante específico devido, dentre outras razões, à relatividade ontológica. Vejamos algumas destas considerações.

O primeiro aspecto que devemos notar é o que chamaremos de processo de abstração. Em prol da simplicidade, objetos físicos podem ser reduzidos à regiões do espaço-tempo ocupadas por eles, e não precisamos parar por aqui: tais regiões nada mais são do que conjuntos de coordenadas do espaço-tempo relativas a algum sistema fixo de coordenadas. E, mais uma vez podemos observar que tais coordenadas de espaço-tempo são quádruplas ordenadas de números reais, e podemos então assumi-las no lugar das coordenadas do espaço-tempo. E ainda podemos reduzir os números reais e as quádruplas ordenadas a conjuntos, ficando com conjuntos de quádruplas ordenadas de números reais que substituiriam os objetos físicos, restando assim nada além de conjuntos puros (isto é, não envolvendo elementos primitivos). Quine observa que:

> We have now looked at three cases in which we interpret or reinterpret one domain of objects by identifying it with part of another domain. In the first example, numbers were identified with some of the classes in one way or another. In the second example, physical objects were identified with some of the place-times, namely, the full ones. In the third example, place-times were identified with some of the classes, namely, classes of quadruples of numbers. In each such case simplicity is gained, if to begin with we had been saddled with the two domains. [5]

Até aqui, vimos como os objetos podem ser assumidos por uma teoria regimentada e, conforme observado nos parágrafos anteriores, podemos reinterpretar um discurso sobre certos objetos em um outro sistema de objetos, desde de que se mantenha fixa a evidência do sistema original. Essa é a tese da relatividade ontológica. Quine observa que, de fato, estamos preservando a estrutura da teoria (o que seja estrutura de uma teoria ainda não nos é claro), enquanto garantimos que as reinterpretações funcionem de modo similar ao fornecer uma função vicária apropriada. Uma função vicária mapeia cada objeto da ontologia original em um objeto da nova ontologia, o que em um arcabouço lógico significaria reinterpretar cada sentença de uma teoria de modo que seja verdadeira para a nova ontologia tal como é verdadeira na interpretação original. Assim, mantemos as condições de buscar em todas as reinterpretações as mesmas evidências e os mesmos resultados teóricos acerca dos objetos de cada ontologia reinterpretada. Quine conclui que a referência é inescrutável, ou seja, não há como dizer nada em absoluto caso nossas palavras se refiram a isso ou aquilo , mas somente se elas se referirem sempre a uma interpretação fixa. A estrutura da teoria está preservada.

Aparentemente, outros problemas parecem surgir. Quando Quine fala de preservação da estrutura e de preservação da verdade das sentenças; uma questão que naturalmente se coloca é: ele está falando de isomorfismo entre estruturas ou de equivalência elementar entre estruturas? Em ambos os casos, equivalência elementar ou isomorfismo, a regimentação não interfere no tribunal da evidência, mas o isomorfismo traz exigências mais fortes do que a equivalência elementar. Estruturas isomorfas são elementarmente equivalentes, mas nem sempre estruturas elementarmente equivalentes são isomorfas. Esse é um aspecto que carece de esclarecimento, sobretudo em nosso caso, uma vez que modelos não standard da aritmética são elementarmente equivalentes ao modelo standard mas não são isomorfos ao modelo standard. Isso é relevante quando analisamos o papel do teorema de Löwenheim-Skolem na tentativa de adoção de uma ontologia pitagórica, isto é, uma ontologia constituída exclusivamente pelos números naturais:

> The conclusion I draw is the inscrutability of reference. To say what objects someone is talking about is to say no more than how we propose to translate his terms into ours; we are free to vary the decision with a proxy function. The translation adopted arrests the free-floating reference of the alien terms only relatively to the free-floating reference of our own terms, by linking the two. [5]

Em uma outra pista, presente na primeira versão do texto *Ontological Reduction and The World of Numbers* [4] Quine escreve “o que justifica a redução de um sistema de objetos a outro é o isomorfismo” , mais tarde o termo isomorfismo foi alterado pelo autor para “o que justifica a redução de um sistema de objetos a outro é a preservação da estrutura relevante". O isomorfismo entre estruturas indica uma similaridade muito forte se comparada, por exemplo, à equivalência elementar entre estruturas. Trata-se, pelo que foi descrito até aqui, de uma mudança muito significativa.

Neste arcabouço lógico há mais um ponto de destaque, o importante teorema de Löwenheim-Skolem, que nos assegura que se uma teoria (de primeira ordem, com uma linguagem enumerável) tem um modelo, então tem um modelo cujo domínio é enumerável. O importante nestas observações é que, pelos dois requisitos apresentados até o momento (preservação do comportamento dos objetos e a preservação das relações que envolvem os objetos) para a redução ontológica, a ideia central seria a descoberta de um modelo. O teorema de Löwenheim-Skolem nos obriga a analisar questões técnicas referentes às noções de teoria e modelo de que Quine faz uso. Tecnicamente, em lógica, um modelo para um conjunto de sentenças de uma linguagem de primeira ordem é uma interpretação dessa linguagem na qual todas essas sentenças são verdadeiras. Já a noção de teoria pode ser estabelecida de duas maneiras. Na primeira, uma teoria pode ser um sistema formal, isto é, uma tripla constituída por uma linguagem, por axiomas e por regras de inferência. No caso de Quine lidamos com teorias de primeira ordem. Por um modelo de uma teoria T, queremos dizer uma estrutura para a linguagem de T na qual todos os axiomas não lógicos de T são verdadeiros. Os teoremas de T são especificados da maneira usual. Na segunda maneira, podemos definir uma teoria de primeira ordem em uma dada linguagem (de primeira ordem, evidentemente) como um conjunto de sentenças dessa linguagem fechado por consequência lógica semântica. Neste caso, os teoremas dessa teoria serão simplesmente os elementos desta teoria. A noção de teoria, tal como usada por Quine no ensaio Relatividade Ontológica, por exemplo, não se enquadra completamente em nenhuma das caraterizações acima:

> To talk thus of theories raises a problem of formulation. A theory, it will be said, is a set of fully interpreted sentences. (More particularly, it is a deductively closed set: it includes all its own logical consequences, insofar as they are couched in the same notation.) But if the sentences of a theory are fully interpreted, then in particular the range of values of their variables is settled. How then can there be no sense in saying what the objects of a theory are? [3]

Como pudemos verificar, as questões lógicas e filosóficas no projeto metafísico de Quine visam uma ontologia cientifica elaborada com o uso da lógica clássica e de aspectos matemáticos da teoria de conjuntos. Entretanto, em muitos casos, o filósofo nos apresenta uma nova interpretação de certos aspectos, fazendo-se necessário esclarecer tal interpretação.

Palavras-chave. Ontologia, Filosofia da Matemática, Lógica.

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Kolmogorov-Veloso Problems, Dialectica Categories and Choice Principles

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Abstract

Blass' paper on questions and answers [1] makes a surprising connection between the second author's Dialectica categories (models of Linear Logic, [2, 3]), Vojtáš' [6] methods to prove inequalities between cardinal characteristics of the continuum (Set Theory) and complexity theoretical notions of problems (and reductions) between these. We recently realized that Kolmogorov's very abstract notion of problem, which is not related to specific complexity issues, can also be intrinsically related to Blass' examples above. Kolmogorov's notion of abstract problem produces an alternative intuitive semantics for Propositional Intuitionistic Logic, an essential component of the celebrated Brouwer-Heyting-Kolmogorov (BHK) interpretation. In this work [4] we connect Kolmogorov's problems to objects of the Dialectica construction, thereby connecting them to Veloso's problems as well [5]. However, we have left clear that Veloso's notions of problems and solutions commit us to a stronger set-theory – namely, the acceptance of the Axiom of Choice within the framework is somehow implied. Nevertheless, we defend that whether this requirement of stronger foundations is a bug or a feature depends on the personal taste and conviction of each researcher. In this talk, we focus on the intrinsic relationship between Veloso's general theory of problems and a number of choice principles (including the Axiom of Choice itself, the Axiom of Countable Choice and the Principle of Dependent Choices).

Dedicated to the memory of Prof. Paulo Veloso (1944-2020)

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The simplicial model of univalent mathematics

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Abstract

In the first half of the last century, category theory arose somehow naturally in advanced studies of homotopy theory. Readily, by the works of Lawvere (see, for instance, [2]) and others, category theory revealed to be of great use for symbolic logic. At the same time, Grothendieck toposes were introduced as an important tool, specially to algebraic geometry. Later on, toposes were also used as tools in logic and set theory, since they are a general framework for building non-standard models of set theories.

Nevertheless, type theory arose in the beginning of the 20th century as a possible solution to Russells paradox in set theory. Throughout the years, it was reformulated to serve as foundation for mathematics based on the theory of recursive functions of Church. Later on, due to its Turing-completeness, it became an universal model of computation. Surprisingly or not, category theory is also deeply related to type theory. In fact, category theory plays an important role as a link tying up logic, computation and mathematics. By the so called Curry-Howard correspondence, types can be seen as propositions in intuitionistic logic.

More recently, a connection between homotopy theory, higher category theory and type theory was discovered, giving rise to what is now known as Homotopy Type Theory (HoTT). In this unifying theory, types are ∞-groupoids! It is an important result of homotopy theory that ∞-groupoids and topological spaces are equivalent.Thus, this resulted in the types as spaces paradigm, which led to many developments both in type theory and in homotopy theory.

Since type theory subsumes so many concepts, why not use types as a primitive concept? By extension, why not use (homotopy) type theory as foundation for mathematical practice? This is the primary goal of the Univalent Foundations book [5]. All the main logical and mathematical concepts are put together, in a very elegant way, using dependent type theory plus the Univalence Axiom (UA). Many results of abstract homotopy theory can be carried on synthetically in HoTT. Moreover, HoTT can be used as a internal language for ∞-groupoids since they all satisfy the UA [4].

We will not go through the features of the book, but we encourage the reader who have not read it yet to take a look. Instead, our intention is to give an explanatory and initial introduction to some of the main ideas behind it and to provide a guide to the comprehension of the simplicial model of dependent type theory established by Voevodsky and published in [1], one of the milestones of the homotopical interpretation of type theory. Such model also introduces a method to avoid coherence problems in dependent type theory using contextual categories and universes, as explored later in [3].

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A diagrammatic solution to a problem on orders

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Abstract

We present a diagrammatic solution to the so called Richard Bird's Problem to find a first-order proof of the following:

Theorem (Richard Bird) For any two preorders X and Y, there exists a preorder Z such that, for any set S, the set of minimum elements w.r.t. Y of the set of minimum elements w.r.t X of the set S is the set of minimum elements w.r.t. Z of the set S.

Our diagrammatic solution is a proof of this result using the Basic Graph Logic (BGL). Bird's theorem is an example of a result whose proof is "far more easily constructed when using the predicate calculus than in case of using the relational calculus", even though the relational calculus is "the symbolism par excellence to tackle such a theorem on relations" [W.H.J. Feijen, *One down for the relational calculus*, Technical Report WF148, 1991]. We present a proof of the same result in our diagrammatic calculus of relations as a *usability* test to BGL.

W.H.J. Feijen reports a proof of R. Bird's theorem above using the relational calculus [3] and another proof using the predicate calculus [4]. Here, we present a diagrammatic proof of the same result, using the Basic Graph Logic (BGL) [1]. We believe that this diagrammatic formalism may bring not only a playful visual approach to the proof but, mainly, it launches an alternative perspective on the heuristic underlying the reasoning applied to prove the result. It seems that for many cases in this context of (binary) relational language, the diagrams of BGL may unfold the proofs smoother than what we see in first order logic or equational algebraic proofs. Moreover, after getting fairly familiar with the diagrams of BGL, based on graphs, it is interesting to see how effective information can be expressed within graphs.

In this three pages abstract, unfortunately, there is no room to include the entire proof of the theorem. So, just for the reader to have a glimpse of BGL-proofs, we will show only one auxiliary result. The interested reader may check the full version of this note [5]. We also omit the formal presentation of BGL-system, which is already done in [1].

Basically, proofs in BGL-system are performed by *graph transformation rules* which mirrors the algebraic behaviour of *relation algebra* operations in the diagrams. Arrows of the graphs are labelled by *relation algebra terms* and the application of the transformation rules are guided by the operators that occurs in the *labels* or by the graph to which one needs to reach in the proof.

To prove the theorem, one presents a relation Z built upon relations X and Y and show that (1) Z is reflexive when X and Y are reflexive, (2) Z is transitive when X and Y are transitive, and (3) $\mathsf{min}_Z S = \mathsf{min}_Y(\mathsf{min}_X S)$, for any set S. Here we show a diagrammatic proof of (2):

(2) Let X and Y be binary transitive relations. Then $Z = X \cap \left(\overline{X}^{-1} \cup Y\right)$ is transitive.

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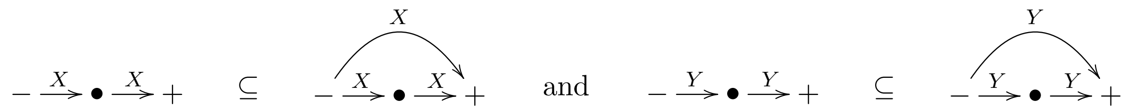
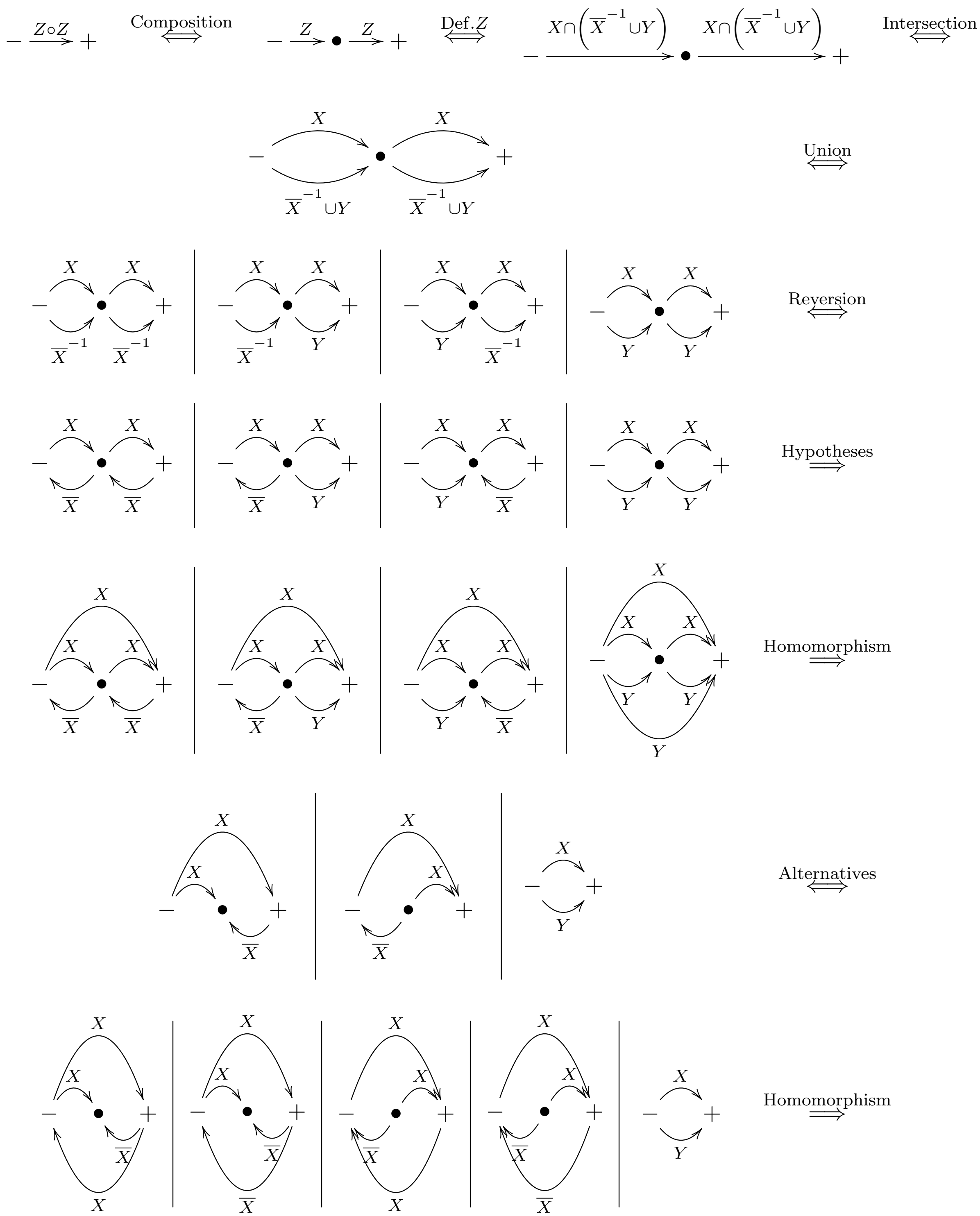
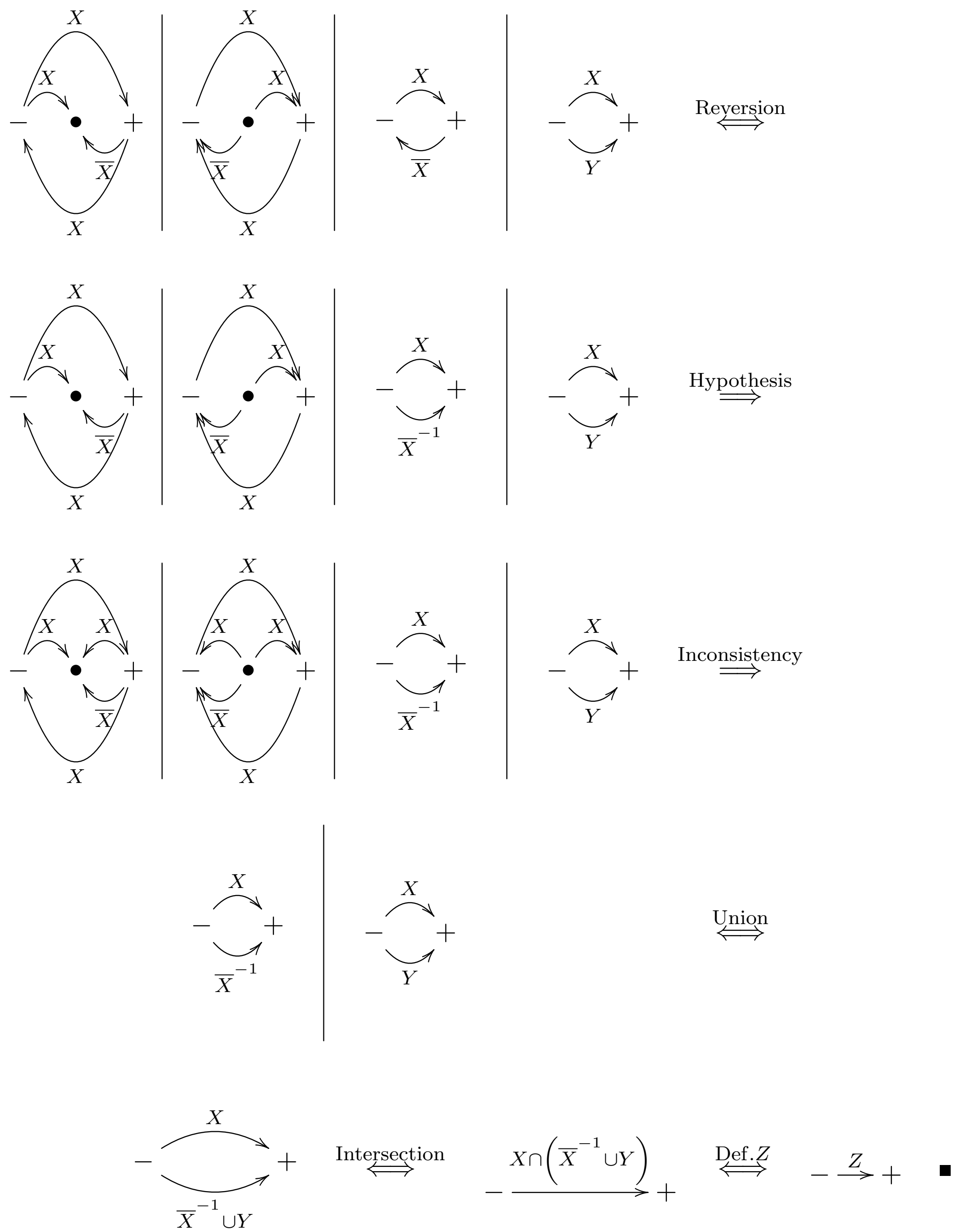
We know that R is transitive iff the composition of R with R is included in R ($R \circ R \subseteq R$). We shall present a diagrammatic proof of $X \circ X \subseteq X, \quad Y \circ Y \subseteq Y \implies Z \circ Z \subseteq Z$, where $Z = X \cap \left(\overline{X}^{-1} \cup Y\right)$.

The diagrammatic versions of the hypotheses are:



Diagrammatic proof of **(2)**:





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ANITA - Analytic Tableau Proof Assistant*

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Resumo

Correntes filosóficas surgiram objetivando a fundamentação da matemática em bases sólidas utilizando a lógica para tal [6]. A escola formalista de Hilbert, por exemplo, buscava a prova da auto-consistência da matemática. Em particular, ela buscava solucionar este problema de uma forma automatizada através de um procedimento (mecânico) que fosse capaz de verificar a consistência das proposições matemáticas.

Como a escola formalista buscava a prova da auto-consistência da matemática, provas em matemática passaram a ser objetos de estudo da própria matemática dando início a uma área conhecida como *Teoria da Prova*. Um dos conceitos centrais desta teoria é o de *sistema dedutivo* que pode ser entendido como um mecanismo que permite a construção de argumentos formais, estabelecendo conclusões a partir de premissas.

Neste contexto, buscando uma melhor análise das estrutura das provas lógicas, foram propostos diversos sistemas dedutivos, tais como os sistemas axiomáticos (a la Hilbert), o sistema de Dedução Natural [3], o sistema de Tableau Analítico [2], dentre outros.

Um curso introdutório de lógica para computação faz parte de quase todos os currículos de graduação em Tecnologia da Informação e Comunicação. Além de ser a base do entendimento de provas matemáticas, a lógica tem um papel de relevo na fundamentação da computação e possui uma vasto espectro de aplicações desde a especificação e verificação de sistemas a aplicações em áreas como banco de dados, inteligência artificial, engenharia de software, dentre outras.

Os sistemas de Dedução Natural e de Tableau Analítico são amplamente utilizados para o ensino de demonstrações e constam em muitos dos livros-texto de Lógica [2, 4, 5].

No campus da Universidade Federal do Ceará em Quixadá, a disciplina Lógica para Computação faz parte dos currículos dos cursos de Sistemas de Informação, Engenharia de Software, Ciência da Computação e Engenharia de Computação como componentes obrigatórios. A disciplina possui um alto índice de reprovação. Para uma melhor assimilação dos conteúdos, é fundamental que os estudantes exercitem e que tenham *feedback* de suas demonstrações.

Há algumas ferramentas para auxiliar o ensino e aprendizagem do conteúdo de sistemas dedutivos. Em [1], apresentamos um assistente de provas, NADIA[1] (*Natural Deduction Proof Assistant*), para o sistema de Dedução Natural, bem como comparamos o sistema com outras ferramentas. NADIA vendo sendo utilizado desde 2021 no campus da UFC em Quixadá.

Para auxiliar o aprendizado dos alunos em demonstração em Tableau Analítico, propomos um assistente de provas, denominado *ANalytic Tableau proof Assistant* (ANITA), que é uma ferramenta computacional que nos permite verificar se uma demonstração está correta ou não. Caso a demonstração não esteja correta, o assistente apresenta os erros encontrados na demonstração.

Este trabalho tem como objetivo apresentar a ferramenta desenvolvida; o contexto em que ela foi utilizada como ferramenta auxiliar no processo de ensino-aprendizagem de Tableau Analítcio em Lógica Proposicional; e, os resultados obtidos com em uma avaliação parcial realizada com 48 alunos matriculados em duas turmas da disciplina de Lógica para Computação durante o semestre letivo de 2022.1. A versão atual do ANITA permite a demonstração de provas em Lógica de Primeira-Ordem. Todavia, como as mesmas não foram utilizadas no período de avaliação da ferramenta, não iremos abordá-la neste artigo.

Os sistemas Axiomático e Dedução Natural permitem demonstrar quando uma fórmula é derivada de um conjunto de fórmulas ($\Gamma \vdash \varphi$). Contudo, nenhum desses métodos nos permite

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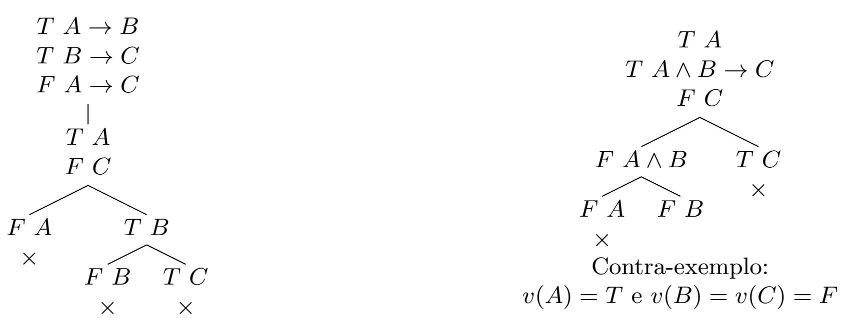
[1]Disponível em `https://sistemas.quixada.ufc.br/nadia/`

inferir que $\Gamma \not\vdash \varphi$. Note que $\Gamma \not\vdash \varphi$ não implica em $\Gamma \vdash \neg\varphi$. O método da Tabela-Verdade é um procedimento de decisão que nos permite provar se $\Gamma \vdash \varphi$ ou $\Gamma \not\vdash \varphi$. Contudo, esse procedimento tem um crescimento no número de linhas exponencial em relação ao número de símbolos proposicionais. O sistema de inferência de **Tableau Analítico** é um método de decisão que não necessariamente gera provas de tamanho exponencial. Tableau é um método de inferência baseado em *refutação*: para provarmos $\Gamma \vdash \varphi$, afirmamos a *veracidade* de Γ e a *falsidade* de φ, na esperança de derivarmos uma *contradição*. Por outro lado, se não for obtida uma contradição, então teremos construído um *contra-exemplo*, i.e., uma valoração que satisfaz Γ e não satisfaz φ.

Para afirmar a veracidade ou falsidade de uma fórmula, o método dos tableaux analíticos marca as fórmulas com os símbolos T para verdade e F para falsidade. O passo inicial na criação de um tableau é marcar todas as fórmulas de Γ com T e a fórmula φ com F. A partir do tableau inicial, utiliza-se regras de expansão do tableau que adicionam novas fórmulas ao final de um ramo (regras do tipo α) ou que bifurcam um ramo em dois (regras do tipo β), como abaixo:

Tipo α	$T\ \varphi \wedge \psi$ | $T\ \varphi$ $T\ \psi$	$F\ \varphi \vee \psi$ | $F\ \varphi$ $F\ \psi$	$F\ \varphi \rightarrow \psi$ | $T\ \varphi$ $F\ \psi$	$T\ \neg\varphi$ | $F\ \varphi$ $F\ \neg\varphi$ | $T\ \varphi$
Tipo β	$F\ \varphi \wedge \psi$ $F\ \varphi$ $F\ \psi$	$T\ \varphi \vee \psi$ $T\ \varphi$ $T\ \psi$	$T\ \varphi \rightarrow \psi$ $F\ \varphi$ $T\ \psi$	

Em cada ramo, uma fórmula só pode ser expandida uma única vez. Um ramo que não possui mais fórmulas para expandir é dito **saturado**. Um ramo que possui uma par de fórmulas $T\ \varphi$ e $F\ \varphi$ é dito **fechado**. Um ramo fechado não precisa mais ser expandido. Um tableau que tem todos os seus ramos fechados é dito fechado, ou seja, $\Gamma \vdash \varphi$. Um ramo saturado e não fechado nos fornece um *contra-exemplo*, ou seja, $\Gamma \not\vdash \varphi$. Na Figura 1a apresentamos a demonstração da transitividade $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$. A Figura 1b exibe uma demonstração em TA, na qual temos um dos ramos (do centro) que se encontra saturado, assim, podemos gerar um contra-exemplo a partir dos valores-verdade dos átomos no ramo.



(a) $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$ (b) $A, A \wedge B \rightarrow C \not\vdash C$

Figura 1: Exemplos de Demonstração em Tableau Analítico

O sistema de Tableau Analítico (TA) usualmente é apresentado por meio de árvores. Todavia, podemos apresentar uma versão de TA em um estilo a la Fitch. No Estilo de Fitch as demonstrações são apresentadas de forma linear e sequencial, na qual cada uma das linhas da prova é numerada, tem uma afirmação e uma justificativa. As justificativas são definidas por serem as premissas ou a conclusão da prova ou pela aplicação de uma das regras do TA.

O assistente de provas ANITA, *ANalytic Tableau proof Assistant*, é uma ferramenta que permite verificar automaticamente a correção de uma demonstração no sistema de Tableau Analítico no estilo a la Fitch. ANITA foi desenvolvido na linguagem Python, pode ser utilizado em *desktop*, ou em uma plataforma *Web*[2]. A ferramenta tem uma área para edição da demonstração em texto simples, uma seção para o resultado da análise da demonstração e os seguintes *links*: *Check*, para verificar a correção da demonstração; *Manual*, para visualizar um documento com as regras e exemplos de demonstração; *Latex*, para gerar o código[3] LaTeX das árvores a partir

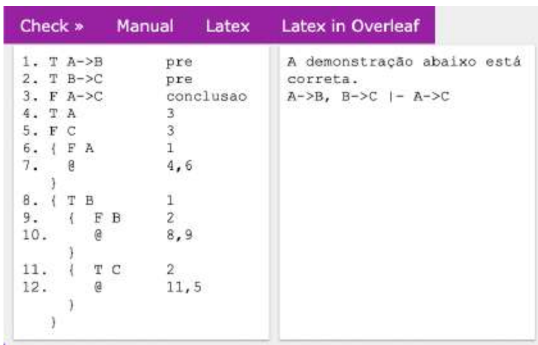
[2]Versão *Web* disponível em: `https://sistemas.quixada.ufc.br/anita/`
[3]Use o pacote *qtree* em seu código LaTeX.

de uma demostração correta; *Latex in Overleaf* para abrir o código diretamente no Overleaf[4].

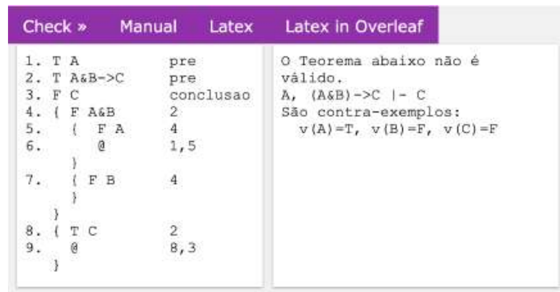
No ANITA, os átomos são escritos em letras maiúsculas (e.g. A, B, C, ...). A Figura 2 apresenta a equivalência dos símbolos da lógica e os que são utilizados no ANITA[5]. As justificativas das Premissas e da Conclusão utilizam as palavras reservadas pre e conclusao, respectivamente. Cada ramo de uma bifurcação da árvore é delimitado por { e }. Um ramo bifurcado pode ser bifurcado novamente por uma regra do tipo beta. Assim, podemos ter aninhamentos de delimitadores { e }. Uma fórmula só pode ser utilizada em uma justificativa em um determinado ponto se essa fórmula aconteceu anteriormente e dentro daquele ramo. As Figuras 3a e 3b apresentam as demonstrações dos exemplos das Figuras 1a e 1b no ANITA.

Símbolo	$\neg$	$\land$	$\lor$	$\rightarrow$	$\forall x$	$\exists x$	$\bot$	bifurcação	$\vdash$
LaTeX	\lnot	\land	\lor	\rightarrow	\forall x	\exists x	\bot	[.]	\vdash
ANITA	∼	&	|	−>	Ax	Ex	@	{}	|−

Figura 2: Equivalência entre os símbolos da lógica, ANITA e LaTeX



(a) $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$



(b) $A, A \land B \rightarrow C \nvdash C$

Figura 3: Exemplos de Demonstração no ANITA

No semestre de 2022.1, utilizamos o NADIA e o ANITA, integrados à plataforma Moodle, na segunda avaliação parcial (AP2), que foi aplicada em laboratório e contou com quatro teoremas a serem demonstrados em Dedução Natural (DN) e quatro em Tableau Analítico (TA), cada quesito valia 1,25. A AP2 foi aplicada em duas turmas. Na turma A, 20 alunos fizeram a AP2 e tiveram média de notas de 7,31 (M), sendo 2,94 (MN) em DN e 4,38 (MA) em TA, com desvio padrão (DP) de 2,93, e 65% (RN) responderam as questões de DN, dos quais 90% acertaram as questões, enquanto 94% (RA) responderam de TA e 93% (AN) dessas acertaram as questões. Os resultados da outra turma estão apresentados na tabela abaixo.

Turma	Alunos	M	DP	MN	DPN	RN	AN	MA	DPA	RA	AN
A	20	7,31	2,93	2,94	2,08	65%	90%	4,38	1,43	94%	93%
B	28	6,44	3,53	2,64	2,03	62%	85%	3,80	1,82	91%	84%

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[4]Overleaf é uma plataforma colaborativa para edição de LaTeX. Disponível em: `http://overleaf.com/`

[5]A ordem de precedência dos quantificadores e dos conectivos lógicos é definida por $\neg, \forall, \exists, \land, \lor, \rightarrow$ com alinhamento à direita. Por exemplo: a fórmula ∼A&B−>C representa a fórmula $(((\neg A) \land B) \rightarrow C)$.

On logical evidence and theory choice

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Abstract

Recent trends in the philosophy of logic, under the title of "anti-exceptionalism", propose that the principles of logic are not a priori, but rather, in naturalist fashion, justified by a posteriori or empirical evidence. An approach favored by anti-exceptionalists is to borrow what they take to be the methodology of justifying theory choice in science, via abduction. Abduction is a mode of inference first proposed by C. S. Peirce, which, together with deduction and induction, are taken to be the sorts of reasoning involved in scientific inquiry. In recent literature, however, this term has lost its original meaning and has come to be employed in the justification of theory selection, as "Inference to the Best Explanation" (IBE). In this sense, the theory which provides the best explanation, according to some theoretical virtues (simplicity, strength, adequacy to the data, and so forth), is the one most likely to be true.

The present contribution argues that logical abductivism is not methodologically sound for logic. First, "abduction" is surveyed from Peirce's original proposal to the current use as synonymous with IBE and it is maintained that these two senses, being different, should not be confused. Second, logical abductivism is presented and it is argued that the sense of abduction employed by anti-exceptionalists is that of IBE, rather than the original sense. Third, it is argued that logical abductivism faces some methodological problems, namely, the logic in the background problem and a problem related to the selection of logical evidence, and thus it is not a suitable approach to theory choice in logic. Fourth, it is argued that logic and science do not share a method of theory choice in a broad sense that the anti-exceptionalist claim it does, as the kind of evidence used to select logical theories does not lend itself to such methods in any straightforward sense.

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Relative expressiveness and stability over language extensions

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Abstract

The concept of notational variance between logics is closely related to the notion of expressive equivalence. It seems reasonable to say that logics differing at most with respect to notation are expressively equivalent. The other way round seems not to hold, however. In [2] it is proposed that notational variance would require what is called *external equivalence*: If logics $\mathcal{L}_1$ and $\mathcal{L}_2$ are notational variants, then every way of extending them by the addition of a new operator # with the same properties in both logics should also result in notational variants. French in [2] gives some logics that would be expressively equivalent but would not satisfy external equivalence, and thus would not be notational variants. One issue here is that the concept of expressive equivalence is not much clearer than notational variance, and there are many different notions of relative expressiveness employed in the literature (*e.g.* in [1], [4], [3] and [5]). Indeed, it would seem reasonable, at least *prima facie*, to require the condition of external equivalence also for the notion of expressive equivalence. The purpose of the talk is to explore the effects the application of this condition would have for some common notions of relative expressiveness.

Keywords. Expressive equivalence, notational variance, formal criteria.

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Dolev-Yao Multi-Agent Epistemic Logic with Communication Actions

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Abstract

In a previous work, the authors have presented a new epistemic logic for reasoning about security protocols based on the Dolev-Yao model [5], the Dolev-Yao Multi-Agent Epistemic Logic [1]. The $\mathcal{S}5_{DY}$ introduces a new semantics based on structured propositions. Instead of building formulas from atomic propositions, they are built from expressions. The latter are any piece of information that can appear in protocols: keys, messages, agents and properties or some combination of this information in pairs, encrypted messages and so forth. This allow us to verify if a malicious user can obtain private messages from a communication network, for example, deriving this information from the messages he received or intercepted. We also provided an axiomatization for this logic and proved it sound and complete.

More recently, we also provided a theorem prover for $\mathcal{S}5_{DY}$ [2]. The method is based on prefixed tableaux [6] and we proved soundness and completeness according to satisfiability results and analytic tableaux conditions, respectively. The set of rules and the termination argument are inspired by Massacci's work [7].

Now our goal is to extend this logic with communication actions. We have been looking for Propositional Dynamic Logic approaches and tableaux calculus for this logic as well. Some initial thoughts are presented below.

The *Dolev-Yao Multi-Agent Epistemic Language with Communication Actions* consists of an enumerable set Φ of propositional symbols, a finite set $\mathcal{A}$ of agents, an enumerable set of keys $\mathcal{K} = \{k_1, \cdots\}$, the Boolean connectives $\neg$ and $\wedge$ and two modalities K_a for each agent a and $\langle m \rangle$.

The *expressions*, *formulae*, *protocols* and *messages* are defined as follows, represented in BNF-notation:

$$E ::= p \mid k \mid (E_1, E_2) \mid \{E\}_k$$

where $k \in \mathcal{K}$ and $p \in \Phi$. $\{E\}_k$ represents the expression E encrypted with the key k.

A formula is defined by the following grammar:

$$\varphi ::= e \mid \top \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid K_a\varphi \mid \langle \pi \rangle \varphi$$

where $e \in E$, $a \in \mathcal{A}$ and π is a protocol.

A protocol is defined as follows:

$$\pi ::= m \mid \pi_1 + \pi_2 \mid \pi_1 ; \pi_2 \mid \varphi?$$

where m is a message.

A message is any expression of the form:

$$m ::= (a, e, b)$$

where $e \in E$ and $a, b \in \mathcal{A}$.

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We call agent a sender and b receiver. For simplicity we define $s_m = a$ (**s**ender of m), $c_m = e$ (**c**ontent of m) and $r_m = b$ (**r**eceiver of m). We are considering the standard abbreviations and conventions: $\bot \equiv \neg\top$, $\varphi \vee \phi \equiv \neg(\neg\varphi \wedge \neg\phi)$, $\varphi \rightarrow \phi \equiv \neg(\varphi \wedge \neg\phi)$, $B_a\varphi \equiv \neg K_a \neg\varphi$ and $[m]\varphi \equiv \neg\langle m\rangle\neg\varphi$.

An initial set of tableaux rules was already designed. First, we present the propositional tableaux rules, for all formulae α and β:

$$\mathrm{R}_\wedge \ \frac{\alpha \wedge \beta}{\begin{array}{c}\alpha\\ \beta\end{array}} \qquad \mathrm{R}_{\mathrm{Dneg}} \ \frac{\neg\neg\alpha}{\alpha} \qquad \mathrm{R}_\wedge^\neg \ \frac{\neg(\alpha \wedge \beta)}{\neg\alpha \quad \neg\beta} \qquad \mathrm{R}_\rightarrow \ \frac{\alpha \rightarrow \beta}{\neg\alpha \quad \beta} \qquad \mathrm{R}_\rightarrow^\neg \ \frac{\neg(\alpha \rightarrow \beta)}{\begin{array}{c}\alpha\\ \neg\beta\end{array}}$$

When rules $\mathrm{R}_\wedge$, $\mathrm{R}_{\mathrm{Dneg}}$ and $\mathrm{R}_\rightarrow^\neg$ are applied, we add the derived subformulae in the same branch of the original formula, while rules $\mathrm{R}_\wedge^\neg$ and $\mathrm{R}_\rightarrow$ split the original branch.

Each tableau will have a different name, so a formula φ in a tableau refutation is unique, identified by (σ, φ), where σ is the *prefix*.

To manage the creation of new tableaux and the addition of new formulae to a previously generated tableau, we denote ρ as the operator which applied on a formula (σ, φ) it will:

- create a new tableau σ', starting with φ, if σ' is not a name for a previously generated tableau subordinated to the branch which φ holds; or
- add φ to the tableau specified by the prefix σ.

So, the rules for $\mathcal{S}5$ are defined as follows:

$$\mathrm{R}_\pi \ \frac{\neg K_a \alpha}{\rho(\mathcal{T}'_a, \neg\alpha)}, \text{ where } \mathcal{T}'_a \text{ is a new tableau, indexed by agent } a$$

$$\mathrm{R}_t \ \frac{K_a\alpha}{\alpha} \qquad \mathrm{R}_4^r \ \frac{\rho(\mathcal{T}''_a, K_a\alpha)}{K_a\alpha}$$

$$\mathrm{R}_4 \ \frac{K_a\alpha}{\rho(\mathcal{T}''_a, K_a\alpha)}, \text{ where } \mathcal{T}''_a \text{ is a previously generated tableau, indexed by agent } a$$

We have rule R_π for the π-formulas (from possibility), while rules R_t, R_4 and R_4^r indicate the correspondence between axioms $K_a\varphi \rightarrow \varphi$, $K_a\varphi \rightarrow K_aK_a\varphi$ and $\neg K_a\varphi \rightarrow K_a\neg K_a\varphi$, respectively, and the properties of accessibility relations (reflexive, transitive and symmetric).

Now we add the $\mathcal{S}5_{DY}$ tableaux rules:

$$\mathrm{R}_{\mathrm{Dec}} \ \frac{\begin{array}{c}\{m\}_k\\ k\end{array}}{m} \qquad \mathrm{R}_{\mathrm{Enc}}^\neg \ \frac{\neg\{m\}_k}{\neg m \quad \neg k} \qquad \mathrm{R}_{\mathrm{Pair}} \ \frac{(m,n)}{\begin{array}{c}m\\ n\end{array}} \qquad \mathrm{R}_{\mathrm{Pair}}^\neg \ \frac{\neg(m,n)}{\neg m \quad \neg n}$$

where $m, \{m\}_k, n, (m,n) \in E$ and $k \in \mathcal{K}$. Those rules indicate the correspondence between our axioms of *encryption*, *decryption* and *pair composition* & *decomposition* ($m \wedge k \rightarrow \{m\}_k$, $\{m\}_k \wedge k \rightarrow m$ and $m \wedge n \leftrightarrow (m,n)$, respectively), and the semantical properties of our valuation function.

For the sake of clarity, we have omitted the prefix when the prefix of the premise is equal to the prefix of the conclusion.

Finally, for the communication actions extension, we have the following rules (inspired by De Giacomo and Massacci's tableaux calculus [3]):

$$\mathrm{R}_{\mathrm{Seq}} \ \frac{\langle\pi_1;\pi_2\rangle\alpha}{\langle\pi_1\rangle\langle\pi_2\rangle\alpha} \qquad \mathrm{R}_{\mathrm{Seq}}^\neg \ \frac{\neg\langle\pi_1;\pi_2\rangle\alpha}{\neg\langle\pi_1\rangle\langle\pi_2\rangle\alpha} \qquad \mathrm{R}_{\mathrm{Test}} \ \frac{\langle\varphi?\rangle\alpha}{\begin{array}{c}\varphi\\ \alpha\end{array}} \qquad \mathrm{R}_{\mathrm{Test}}^\neg \ \frac{\neg\langle\varphi?\rangle\alpha}{\neg\varphi \quad \neg\alpha}$$

$$\mathrm{R}_{\mathrm{Choice}} \ \frac{\langle\pi_1+\pi_2\rangle\alpha}{\langle\pi_1\rangle\alpha \quad \langle\pi_2\rangle\alpha} \qquad \mathrm{R}_{\mathrm{Choice}}^\neg \ \frac{\neg\langle\pi_1+\pi_2\rangle\alpha}{\begin{array}{c}\neg\langle\pi_1\rangle\alpha\\ \neg\langle\pi_2\rangle\alpha\end{array}} \qquad \mathrm{R}_{\langle A\rangle} \ \frac{\sigma:\ \langle A\rangle\alpha}{\sigma.A.n:\ \alpha}, \text{ with } \sigma.A.n \text{ new in the branch}$$

$$\mathrm{R}_{\langle A\rangle}^\neg \ \frac{\sigma:\ \neg\langle A\rangle\alpha}{\sigma.A.n:\ \neg\alpha}, \text{ with } \sigma.A.n \text{ already present in the branch}$$

where A is an atomic program.

This is a work in progress and this is what we have done so far. Currently, we are defining the set of tableaux rules and their conditions. For instance, the condition of rule $R^{\neg}_{\langle A \rangle}$ is still being discussed with respect to a possible serial accessibility relation. We are also studying the necessity of perfect recall in our axiomatization. In the future, we will work on giving semantics, axioms and inference rules for this extension and proving it sound and complete. We are also planning to implement its tableaux system in LoTREC [4] and provide the soundness, completenesss and termination argument proofs.

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Approximating Łukasiewicz Infinitely-valued Logic via Polyhedral Semantics*

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Abstract

We present *polyhedral semantics*, a semantic approximation of Łukasiewicz infinitely-valued logic ($Ł_\infty$) [1]. As $Ł_\infty$ is an expressive multi-valued propositional logic whose decision problem is NP-complete, we hope to provide a tractable approximation for this problem providing a tractable family of multi-valued logics over the same language as $Ł_\infty$.

To the best of our knowledge, this is the first approximating system presented for multi-valued logics. The idea of *approximate entailment* has been proposed and developed both as a means of modeling the reasoning of an agent with limited resources and as a means to convey tractable reasoning to intractable systems [2–4]. Even if all approximate reasoning systems consist of non-classical logics, the vast majority of approximate reasoning systems have classical propositional logic as its target.

Here we propose to use parameterized polyhedral semantics as a system that approximates non-classical multi-valued logic $Ł_\infty$. The approximations are based on a Mixed Integer Linear Programming (MILP) presentation of $Ł_\infty$-semantics, which combines a set of continuous inequalities (the linear programming part) with a set of binary bits (the integer part), obtaining a functional definition of $Ł_\infty$-semantics. Polyhedral semantics is established by allowing the binary $\{0,1\}$-variables in the MILP presentation to take continuous values in $[0,1]$. The resulting system is a set of inequalities and thus tractable. Approximation steps are obtained by forcing a restricted number of the relaxed variables back to take values in $\{0,1\}$.

For example, for the $Ł_\infty$-disjunction, let $y_{\varphi\oplus\psi} = v(\varphi\oplus\psi)$, then by considering an extra symbol $b_{\varphi\oplus\psi}$, the following restrictions imposed on $y_{\varphi\oplus\psi}$ guarantee $v(\varphi\oplus\psi) = \min(1, v(\varphi)+v(\psi))$:

$$\begin{aligned} b_{\varphi\oplus\psi} &\in \{0,1\} \\ b_{\varphi\oplus\psi} &\leq y_{\varphi\oplus\psi} \leq 1 \\ y_\varphi + y_\psi - b_{\varphi\oplus\psi} &\leq y_{\varphi\oplus\psi} \leq y_\varphi + y_\psi \end{aligned} \tag{1}$$

When $b_{\varphi\oplus\psi} = 0$, $y_{\varphi\oplus\psi} = y_\varphi + y_\psi \leq 1$; and when $b_{\varphi\oplus\psi} = 1$, $y_{\varphi\oplus\psi} = 1 \leq y_\varphi + y_\psi$. So $y_{\varphi\oplus\psi} = \min(1, y_\varphi + y_\psi)$. Importantly, when we relax $b_{\varphi\oplus\psi}$ to take any value in $[0,1]$, the MILP problem becomes a tractable linear program and a set of inequalities like (1) defines a polyhedron.

Theorem 1 *The decision procedures for satisfiability in polyhedral semantics is tractable.* □

An approximation process to decide satisfiability for a set Φ of $Ł_\infty$-formulas is parameterized by a sequence of sets of relaxed variables

$$\emptyset = R_0 \subset R_1 \subset \cdots \subset R_i \subset \cdots \subset R_{\text{end}};$$

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at each step i, one has to consider 2^i relaxed linear systems. If all those systems are unsolvable, then Φ is unsatisfiable and the decision process terminates; otherwise nothing can in $Ł_\infty$ be inferred about Φ. If no unsatisfiability decision is reached, the process proceeds until R_{end} is reached and a final decision is made. This process is called a *logical approximation from below* by [3].

Lemma 1 (Soundness of $Ł_\infty$-approximation) *Suppose we are at step i in a satisfiability approximation from below for a set of formulas Φ, such that all the 2^i associated relaxed linear systems are unsolvable. Then Φ is unsatisfiable in $Ł_\infty$.* □

Let $Ł_\infty(\Psi)$ be a multi-valued logic system evaluated with polyhedral semantics except that the relaxed variables associated to $\psi \in \Psi$ are restricted to the values in $\{0, 1\}$. It is immediate that for each R_i there exists a corresponding $Ł_\infty(\Psi_i)$.

Finally, we have the main result for approximations employing polyhedral semantics, namely that up to a certain level in the approximation process the approximation is tractable.

Theorem 2 *Let Φ be a set of formulas such that the number of subformulas in Φ is n. Suppose approximate logic $Ł_\infty(\Psi)$ is such that $|\Psi| = O(\log n)$. Then the decision of α/α^+-satisfiability in $Ł_\infty(\Psi)$ has complexity time polynomial in n.* □

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Uma análise da noção de indecidibilidade presente nos teoremas de incompletude de Gödel e no problema da parada

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Resumo

As vertentes logicista e formalista da filosofia da matemática contemporânea possuíam um objetivo em comum: fundamentar a aritmética em um sistema formal. Caso esse intento se cumprisse, seríamos levados a acreditar, ou ter pelo menos a expectativa, que é possível resolver qualquer problema aritmético por meio de uma máquina de Turing. O problema da decisão de Hilbert está diretamente relacionado com essa questão, que permeia o debate a respeito de mentes e máquinas. Podemos resumir o problema da Decisão (*Entscheidungsproblem*), na seguinte pergunta:

> Existe um procedimento mecânico capaz de determinar se todos os enunciados verdadeiros da matemática podem ou não ser provados?

De acordo com o programa formalista de Hilbert, toda matemática poderia ser compreendida como um conjunto de fórmulas verdadeiras, que são deduzidas de axiomas básicos. Assim, diante de qualquer enunciado matemático, uma máquina de Turing alimentada com os axiomas básicos e com a sintaxe, ou seja, as regras de manipulação dos símbolos e axiomas, poderia decidir se o enunciado matemático é verdadeiro ou falso.

A máquina de Turing, por seu turno, é um constructo teórico, que podemos compreender como um sistema formal. Toda solução de qualquer problema matemático, é encontrada depois de percorrermos um número finito de passos. Levando em consideração o conceito de algoritmo, apresentado por Turing, como um processo ordenado de regras usado para resolver um problema. As soluções de problemas são apresentadas depois de um algoritmo ter sido efetuado, isso é um procedimento sistemático finito.

O problema da fundamentação da matemática, preocupação comum entre logicistas e formalistas, pode ser reduzido ao problema da consistência da aritmética. A busca da consistência da aritmética em um sistema lógico-formal exige que esse sistema seja finitamente descritível, consistente, completo e capaz de representar enunciado acerca dos números naturais.

O problema da decidibilidade indaga se diante um enunciado da aritmética podemos, com base em um algoritmo, afirmar se tal enunciado é verdadeiro ou falso. O seguinte enunciado: "a soma de dois números ímpares é sempre um número par" é um exemplo de enunciado aritmético. Precisamos de um algoritmo, ou seja, um procedimento finito que pode ser computado e, após todos os passos realizados, devemos ter a resposta a respeito do enunciado. No caso do programa de Hilbert isso deve ser feito em um sistema formal que abrange a aritmética.

Kurt Gödel, em 1931, com seus Teoremas de Incompletude, mostra as dificuldades em formalizar a aritmética. Pressupondo a consistência da aritmética fundada em um sistema lógico-formal, construído de maneira recursiva, tal teoria se mostrará incompleta, ou seja, haverá enunciados aritméticos que não são demonstráveis.

Gödel busca expressar paradoxos de sentenças autorreferentes na linguagem do sistema lógico formal, utilizando no caso o *Principia Mathematica* de Russell e Whithead. Procurando uma exceção para a completude almejada por Hilbert, deveria existir, segundo Gödel, uma sentença não demonstrável no sistema formal. Reescreve então o paradoxo do mentiroso da seguinte maneira: "este enunciado não é demonstrável".

No entanto, ao codificar, a sentença "este enunciado não é demonstrável" passa a expressar na linguagem aritmética um enunciado metamatemático autorreferente. O cerne do problema

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é que o enunciado “este enunciado não é demonstrável” é demonstrável. Isso implica uma inconsistência no esquema de prova.

Se o enunciado “este enunciado não é demonstrável”, não é demonstrável, temos então um enunciado que pode ser construído, mas não pode ser demonstrado, isso implica que o sistema formal utilizado para provar enunciados é incompleto. O primeiro teorema de incompletude de Gödel explicita que, se um sistema lógico-formal é usado para descrever a aritmética, tal sistema deve ser necessariamente incompleto para ser consistente.

A formalização é incompleta para todos sistemas onde é possível expressar o enunciado de Gödel. Um enunciado de Gödel é um enunciado que não é passível de prova mesmo que sejam utilizadas todas as regras do sistema. A verdade de tal enunciado só pode ser afirmada externamente ao sistema, isso configura o conteúdo do segundo teorema de incompletude de Gödel.

O enunciado “a aritmética é consistente” não é demonstrável em um sistema formal que represente a aritmética. Se pressupormos a consistência da aritmética, essa não é demonstrável em um sistema formal que representa a própria aritmética. De maneira análoga, ao considerarmos um programa H, que opera de maneira equivalente ao sistema formal. Esse programa H é capaz de decidir a respeito dos programas em máquinas de Turing param e que não param. Deve existir então um programa P, análogo ao enunciado de Gödel e dados de input I, tal que H não é capaz de decidir se P para ou não se alimentado com os dados I.

Essa analogia nos mostra que existem enunciados aritméticos que não são formalizáveis em sistemas lógico-formais e explicita uma proximidade da tese formalista da filosofia da matemática com premissas presentes na teoria funcionalista na filosofia da mente.O problema da parada e os resultados limitativos apresentados pelos teoremas de incompletude estão relacionados na chamada disjunção de Gödel, a qual podemos enunciar da seguinte maneira:

> Ou a mente humana não é equivalente a uma máquina de Turing, ou existem sentenças aritméticas verdadeiras absolutamente indemonstráveis

Essa disjunção contesta a tese computacionalista a respeito da mente humana e fala a respeito da noção de demonstrabilidade. A relação entre a capacidade de demonstração de teorias aritméticas, explicitada pelos teoremas de incompletude, e o problema da parada, relacionados no âmbito da disjunção de Gödel constituem assim o principal objeto de estudo do presente trabalho.

Palavras-chave. Formalismo, Funcionalismo, Indecidibilidade, Problema da Parada, Teoremas de Incompletude de Gödel.

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The strength of monadic first-order theories

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Abstract

A philosophically relevant question about first-order theories concerns its strength, and an explanation of such notion can be given in terms of interpretations[1]. The concept of interpretation admits a series of refinements. In this work, we will focus on a particular kind of mutual relation of interpretation, namely, bi-interpretation[2]. Assuming that bi-interpretable theories are "essentially the same" (see [1]), we motivate and develop the central theme of this work: The tightness of a theory.

A theory T is tight if and only if no pair of different and deductively closed extensions of T are bi-interpretable. This means that whenever one finds two bi-interpretable extensions T_1 and T_2 of a tight theory T, one may conclude that these theories have precisely the same theorems and they are, therefore, different axiomatizations of the same concepts.

Recently, it has been verified that several theories relevant to the foundations of mathematics are tight. For example, Peano arithmetic, second-order arithmetic, ZF set theory, and Kelley-Morse theory of classes are tight [2], [5]. The set theories obtained from ZF by excluding the axiom schema of replacement or excluding the power-set axiom are not tight [3].

This is a work in progress and our main objective is to present a proof that all first-order monadic theories, that is, first-order theories whose signature has only unary predicate symbols, are tight.

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[1] The notion of interpretation between theories is quite general and widespread in foundational studies. For a precise presentation of the notion of interpretation here considered see [4, §4.7].

[2] Two theories T and T' are bi-interpretable if there is an interpretation I of T in T' and an interpretation J of T' in T such that their compositions are isomorphic to the identity in the appropriated theory; a precise definition of isomorphism between interpretations can be found in [4, §9.5]. The formulation of bi-interpretation presented in this work can be rephrased in terms of model theory and category theory.

The epic history of a name: the role of Newton da Costa and Francisco Miró Quesada in the baptism of paraconsistent logics

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Abstract

The study of inconsistent but non-trivial theories, and of the deductive systems underlying such theories, was practiced for some time, from the 1960's into the 1970's, without a suitable name being attributed to it. Until an appropriate name was finally proposed, theorists involved in the investigation of these systems simply referred to them as 'logics of inconsistent formal systems'. The birth certificate of paraconsistent logic was drawn up in a letter from the Peruvian philosopher Francisco Miró Quesada Cantuarias (1918–2019), the proposer of the name, to Newton da Costa (1929), one of the creators of modern paraconsistent logics (see [1]). In the abundant correspondence between them, the letter dated September 29, 1975, is especially remarkable. In this notable letter, Miró Quesada begins by expressing great contentment over da Costa's having invited him to come the following year to the University of Campinas (Unicamp), in Campinas, São Paulo State, Brazil, to participate in the Third Latin American Symposium on Mathematical Logic (III SLALM). However, Miró Quesada was even more satisfied to be able to respond to his friend's request that he had found a name for the logics of inconsistent and non-trivial formal systems. The aim of one of our general research projects, to which this paper belongs, consists in studying how a truly paraconsistent perspective was constituted in Western thought, as well as how logical principles, rules, and systems have expressed the various contemporary concepts of paraconsistency. During the development of this project, we have done careful research in the documents donated by Newton da Costa to the Historical Archives of the Centre for Logic, Epistemology and the History of Science, and which constitute the "Newton da Costa Trust". There we found the precious letter addressed by Francisco Miró Quesada to Newton da Costa (see [4]), which we have mentioned at certain academic events in which we have participated. A facsimile of the letter was published for the first time in the doctoral dissertation of Evandro Luís Gomes, *Sobre a história da paraconsistência e a obra de da Costa: a instauração da lógica paraconsistente* (*On the history of paraconsistency and da Costa's work: the establishment of paraconsistent logic*, in Portuguese), defended at the University of Campinas (Unicamp) in December of 2013 under the supervision of Itala M. Loffredo D'Ottaviano (see [2, pp. 609–610]). The facsimile also appears in the book *Para além das Colunas de Hércules: uma história da paraconsistência de Heráclito a Newton da Costa* (*Beyond the Column of Hercules: a history of paraconsistency from Heraclitus to Newton da Costa*, in Portuguese; see [3, pp. 610–1]). In this paper, we present and analyse the main known historical events concerning the creation of the word 'paraconsistent', as well as its introduction as the name for inconsistent but non-trivial formal systems. First, we present the famous correspondence between Francisco Miró Quesada and Newton da Costa, with the facsimile of Miró Quesada's pivotal letter of September 29th, 1975. Second, we describe how the terms 'paraconsistent' and 'paraconsistent logic' were introduced by Miró Quesada, and proposed to the logical academic community during the Third Latin

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American Symposium on Mathematical Logic (III SLALM), organized by Ayda Ignez Arruda and held at the University of Campinas. Third, we discuss the etymological roots of the term 'paraconsistent', previously proposed by Miró Quesada to Newton da Costa. In conclusion, we present our final remarks, claiming that Francisco Miró Quesada ineradicably left his mark on the history of paraconsistency and paraconsistent logic.

Keywords. history of logic, non-classical logics, paraconsistent logic, Newton Carneiro Affonso da Costa, Francisco Miró Quesada Cantuarias, Third Latin American Symposium on Mathematical Logic, University of Campinas (Unicamp).

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Informação e Significado

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Resumo

Uma abordagem muito conhecida acerca do significado determina o seguinte: o significado de uma proposição corresponde às suas *condições de verdade*, ou seja, saber como os fatos e eventos devem estar configurados para que a proposição seja verdadeira. Wittgenstein apresenta essa noção de significado no Tractatus: "4.024 - Compreender uma proposição é saber o que ocorre, caso ela fôr verdadeira" [1], p. 56. Essa concepção verocondicional do significado também é coerente com a Gramática de Montague, uma vez que, neste contexto, linguagens naturais e linguagens formais podem ser tratadas de maneiras semelhantes. Uma concepção alternativa a essa é a de Jerry Fodor [11]. Para ele, o significado de uma frase declarativa consiste na expressão da linguagem de pensamento que ele corresponde. Uma das dificuldades da concepção vericondicional de significado é a formulação de um modelo empírico adequado para o processo de compreensão de significado por parte de falantes humanos [9]. Nesta apresentação será iniciada uma discussão acerca desse modelo a partir da junção entre uma teoria do significado e a teoria da informação. Vários importantes autores contribuíram para a teoria geral do significado: Peirce, Frege, Russell, Wittgesntein, Quine, Kripke, entre outros. A aproximação entre o estudo filosófico do significado e o estudo filosófico da informação foi introduzida por alguns pesquisadores e filósofos a saber: Carnap e Bar-Hillel [2] [3], Mackay [4], Hintikka [5], Stonier [10] e Menant [6]. Por exemplo, para Menant, significado e informação são conceitos fortemente conectados quando se trata de contextos humanos. Para ele, informações significativas surgem de sistemas que são submetidos a algumas restrições. Nesse caso, "um significado é uma informação bem formada que é criada por um sistema submetido a uma restrição quando ele recebe uma informação incidente que tem uma conexão com a restrição. O significado é formado pela conexão existente entre a informação recebida e a restrição do sistema. A função da informação significativa é participar da determinação de uma ação que será implementada para satisfazer a restrição do sistema." [6], p. 197. É notório que a teoria da informação, tal como foi concebida por Claude Shannon [8], não apresentou uma definição precisa de informação e tampouco desenvolveu a sua parte semântica. Já na década de 50 do século XX, Carnap e Bar-Hillel propuseram uma complementação semântica para a teoria da informação [2] [3]. Essa proposta foi generalizada para a lógica de primeira ordem por Hintikka em 1970 [5]. Gorsky estendeu a teoria de Carnap e Bar-Hillel para contextos não-clássicos usando como base algumas instâncias da lógica paraconsistente [7]. Seguindo essa linha, O presente trabalho tem como objetivo aprofundar o entendimento do papel do significado e da informação para a compreensão em um sistema de comunicação. Para isso serão utilizadas estruturas semelhantes às que Shannon utilizou para a quantificação da informação. Essa caracterização possibilita uma interpretação epistêmica da teoria. Da mesma maneira que pensamos a escolha de sequências de sinais dentro de um universo de possibilidades, podemos pensar o significado como sendo um dos possíveis significados em um universo de possibilidades. Dessa forma, será possível apresentar um modelo para a noção de compreensão de discurso entre agentes de comunicação.

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Finite and analytic proof systems for non-finitely axiomatizable logics

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The characterizing properties of a proof-theoretical presentation of a given logic may hang on the choice of proof formalism, on the shape of the logical rules and of the sequents manipulated by a given proof system, on the underlying notion of consequence, and even on the expressiveness of its linguistic resources and on the logical framework into which it is embedded.

In [5], the received notions of consequence relation, non-deterministic logical matrices and Hilbert-style systems were generalized to the context of two-dimensional logics [3], which we detail in what follows. Let Σ be a propositional signature, P be a denumerable set of propositional variables and $L_\Sigma(P)$ be the language over Σ generated by P. We refer to each 2×2 tuple $\left(\begin{smallmatrix}\Phi_{\mathsf{И}} & \Phi_{\mathsf{⅄}} \\ \Phi_{\mathsf{Y}} & \Phi_{\mathsf{N}}\end{smallmatrix}\right)$ in $\mathcal{P}(L_\Sigma(P))$ as a B-*statement*, of which $(\Phi_{\mathsf{Y}}, \Phi_{\mathsf{N}})$ is the *antecedent* and $(\Phi_{\mathsf{⅄}}, \Phi_{\mathsf{И}})$ is the *succedent*. The sets involved in the latter two pairs are called *components*. A *two-dimensional logic* (alternatively, a B-*consequence relation*) is a collection $\frac{\cdot}{\cdot}|\frac{\cdot}{\cdot}$ of B-statements respecting:

(O2) if $\Phi_{\mathsf{Y}} \cap \Phi_{\mathsf{⅄}} \neq \varnothing$ or $\Phi_{\mathsf{N}} \cap \Phi_{\mathsf{И}} \neq \varnothing$, then $\frac{\Phi_{\mathsf{И}}}{\Phi_{\mathsf{Y}}}|\frac{\Phi_{\mathsf{⅄}}}{\Phi_{\mathsf{N}}}$

(D2) if $\frac{\Psi_{\mathsf{И}}}{\Psi_{\mathsf{Y}}}|\frac{\Psi_{\mathsf{⅄}}}{\Psi_{\mathsf{N}}}$ and $\Psi_\alpha \subseteq \Phi_\alpha$ for every $\alpha \in \{\mathsf{Y}, \mathsf{⅄}, \mathsf{N}, \mathsf{И}\}$, then $\frac{\Phi_{\mathsf{И}}}{\Phi_{\mathsf{Y}}}|\frac{\Phi_{\mathsf{⅄}}}{\Phi_{\mathsf{N}}}$

(C2) if $\frac{\Omega_{\mathsf{Ƨ}}^{c}}{\Omega_{\mathsf{S}}}|\frac{\Omega_{\mathsf{S}}^{c}}{\Omega_{\mathsf{Ƨ}}}$ for all $\Phi_{\mathsf{Y}} \subseteq \Omega_{\mathsf{S}} \subseteq \Phi_{\mathsf{⅄}}^{c}$ and $\Phi_{\mathsf{N}} \subseteq \Omega_{\mathsf{Ƨ}} \subseteq \Phi_{\mathsf{И}}^{c}$, then $\frac{\Phi_{\mathsf{И}}}{\Phi_{\mathsf{Y}}}|\frac{\Phi_{\mathsf{⅄}}}{\Phi_{\mathsf{N}}}$

A two-dimensional logic may also satisfy two-dimensional versions of the properties of substitution-invariance and finitariness. A *non-deterministic* B-*matrix over* Σ, or simply Σ-*nd*-B-*matrix*, is a structure $\mathfrak{M} := \langle \mathbf{A}, \mathsf{Y}, \mathsf{N} \rangle$, where $\mathbf{A}$ is a (partial) non-deterministic Σ-algebra [1, 2], $\mathsf{Y} \subseteq A$ is the set of *designated values* and $\mathsf{N} \subseteq A$ is the set of *antidesignated values* of $\mathfrak{M}$. Every (finite) Σ-nd-B-matrix canonically determines a (finitary) two-dimensional logic denoted by $\frac{\cdot}{\cdot}|\frac{\cdot}{\cdot}{}_{\mathfrak{M}}$ (the B-*entailment determined by* $\mathbb{M}$). With respect to $\textsc{Set}^2$-$\textsc{Set}^2$ (or *two-dimensional*) H-systems, first introduced in [5], we define a *(schematic)* $\textsc{Set}^2$-$\textsc{Set}^2$ *rule of inference* $R_{\mathfrak{s}}$ as the collection of all substitution instances of the $\textsc{Set}^2$-$\textsc{Set}^2$ statement $\mathfrak{s}$, called the *schema* of $R_{\mathfrak{s}}$. Each $r \in R_{\mathfrak{s}}$ is said to be a *rule instance of* $R_{\mathfrak{s}}$. In a proof-theoretic context, rather than writing the B-statement $\left(\begin{smallmatrix}\Phi_{\mathsf{И}} & \Phi_{\mathsf{⅄}} \\ \Phi_{\mathsf{Y}} & \Phi_{\mathsf{N}}\end{smallmatrix}\right)$, we shall denote the corresponding rule by $\frac{\Phi_{\mathsf{Y}} \parallel \Phi_{\mathsf{N}}}{\Phi_{\mathsf{⅄}} \parallel \Phi_{\mathsf{И}}}$. A *(schematic)* $\textsc{Set}^2$-$\textsc{Set}^2$ *H-system* $\mathfrak{R}$ is a collection of $\textsc{Set}^2$-$\textsc{Set}^2$ rules of inference. $\textsc{Set}^2$-$\textsc{Set}^2$ *derivations* are as in the $\textsc{Set}$-$\textsc{Set}$ H-systems, but now the nodes are labelled with pairs of sets of formulas, instead of a single set. When applying a rule instance, each formula in the succedent produces a new branch, with the formula going to the same component in which it was found in the rule instance (see Figure 1). Building on these notions of two-dimensional logics, matrices and H-systems, one may find in [5] a generalization of the property of sufficient expressiveness and of the axiomatizability result presented in [4,7,8], namely, that every sufficiently expressive (finite) B-matrix is axiomatized by a (finite) $\textsc{Set}^2$-$\textsc{Set}^2$ analytic H-system.

Taking advantage of the above generalizations, in this work we introduce a recipe for cooking up a B-matrix by the combination of two (partial) non-deterministic logical matrices and show that

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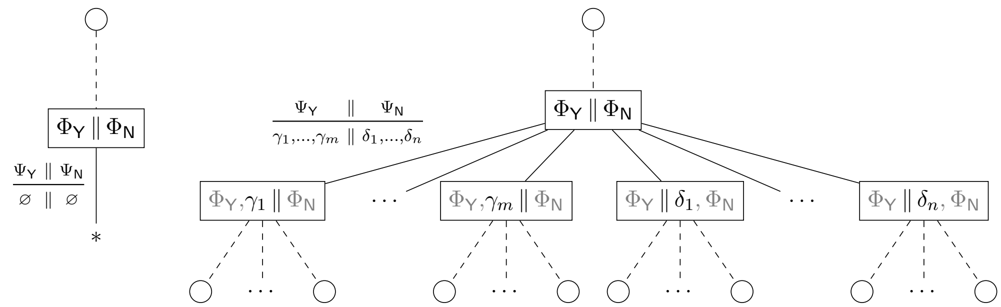

Figure 1: Graphical representation of finite $\mathfrak{R}$-derivations. We emphasize that, in both cases, we must have $\Psi_{\mathsf{Y}} \subseteq \Phi_{\mathsf{Y}}$ and $\Psi_{\mathsf{N}} \subseteq \Phi_{\mathsf{N}}$ to enable the application of the rule.

such a combination may result in B-matrices satisfying the property of sufficient expressiveness, even when the input matrices are not sufficiently expressive in isolation. We will use this result to show that one-dimensional logics that are not finitely axiomatizable may inhabit finitely axiomatizable two-dimensional logics, becoming, thus, finitely axiomatizable by the addition of an extra dimension. The said procedure will be illustrated using a well-known logic of formal inconsistency called **mCi**. We will first prove that this logic is not finitely axiomatizable by a one-dimensional (generalized) Hilbert-style system. Then, taking advantage of a known 5-valued non-deterministic logical matrix for this logic, we will combine it with another one, conveniently chosen so as to give rise to a B-matrix that is axiomatized by a two-dimensional Hilbert-style system that is both finite and analytic. Details may be found at [6].

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Physical computational by manifolds

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Abstract

There are interpretation of Church's thesis regarding the machine model proposed by Turing to represent the intuitive notion of computability that go beyond its real meaning. What Turing proposed was a model that has the power to simulate all idealized mathematical processes [7].

Turing does not deny that other models of computation can calculate things that Turing machines cannot. He does not even deny that any physically realizable machine could exceed the power of a Turing machine. He simply equates the power of Turing machines with the somewhat vague notion of a methodical mathematician.

Copeland [4] presents a valuable account of many of the main articles and books that distorted the Church-Turing Thesis and examines some of the effects of this on computational theory.

There is a substantial difference between processes performed by idealized mathematicians and mathematically usable processes. Turing machines simulate the former type. In contrast, the second type escapes the mathematical interpretation of computability since deals with the physical nature of the universe related to the theory.

The machines that consider the physical scope belong to the so-called physical model of Turing machines.

Gandy [5] presents an essential link between the Machines in the Turing formal thesis and the Physics thesis by providing an argument for Thesis M: "What can be calculated by a physical device under restricted hypotheses is Turing computable.".

In short, Gandy's Theorem states that what can be calculated by a physical device satisfying F, T, I, V and Q is Turing computable, where:

(F) Homogeneity of Space: If τ is a translation of A, a region of space, then τA has the same state set of A.

(T) Time homogeneity: The mapping from $ST_t(A)$ to $ST_{t+\delta}(A)$ is independent of t, for each region A.

(I) Limited information density: If A is an abundant region, then $ST_t(A)$ is a finite set.

(V) Limited information propagation speed: There is T such that, for any region A and time t, $ST_t(A)$ depends only on $ST_{t+T}(A')$, where A' is the region with radius 1 around A.

(Q) Quiescence: Any region A has a state q_A, the quiescent state of A. If A is in the quiescent state, then, if $B \subset A$, B is in the q_B state for any B.

However, Gandy's notion of determinism, and his description of discrete deterministic machines, are not broad enough to apply to discrete deterministic mechanical computing.

Also, according to [3], Gandy's notion of a "mechanical device" is quite narrow. Gandy focuses on discrete deterministic systems and leaves out interesting cases of ideal physical computing systems. We remind the reader that quantum systems are discrete and within the scope of Gandy's approach. See [1] that considers an extension of Gandy's approach to quantum mechanical systems and quantum computing.

What does seem to be remaining to consider is infinite computing. Although infinite running physical systems consume infinite energy, they cannot consume infinite energy in a finite time. Thus, the consumption of energy or information should be locally finite. We also remember that information can be regarded as the computing counterpart of energy.

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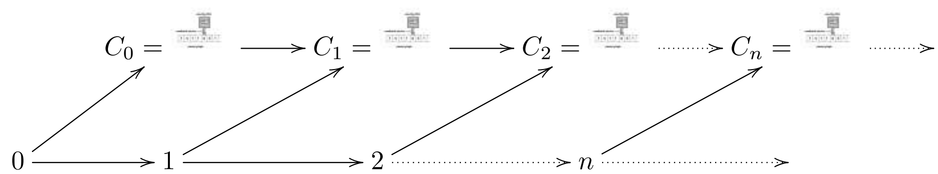
In mathematics, manifolds are mostly considered as an adequate and complete way to deal with the description of global features by means of harmonic/uniform/consistent local features integration.

In this work, we present a computational model based on manifolds and show that A is computable if and only if there is a computational manifold that represents A, where A is a possible non-terminating discrete dynamical system .

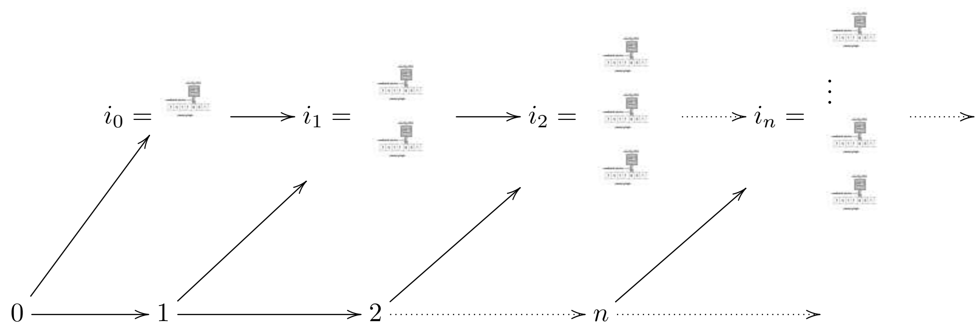
Considering that effective computation must be locally finite, the use of manifolds shows to be an excellent choice for representing computation.

From the categorical point of view [6], functions computable everywhere can be represented in $FinSets^{\omega}$. Consider a functor C that maps n to the machine configuration C_n at step n:



1. Step 0 is mapped into a chain of inclusions: $i_0(0) \hookrightarrow i_0(1) \hookrightarrow i_0(2) \cdots i_0(n) \hookrightarrow \cdots$;
2. This chain represents all possible input configurations;
3. A functor maps to each step n the functor F_n representing the evolving chains.

The picture for the input functor is:

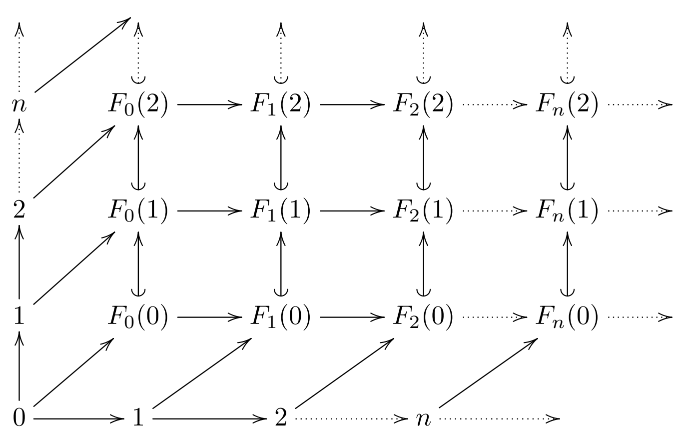


Step 0 is mapped into a chain of inclusions. This chain represents all possible input configurations. A functor maps to each step n the functor F_n representing the evolving chains. [1] The big picture is below.

In details, given a fixed Turing machine T, and considering a linear order on all the possible initial configurations (tapes) determined by each input in $\{d_0, d_1, \ldots\}$, i_0 is the initial configuration of T set to the input data d_0 and $i_{n+1} = i_n \cup \{d_{n+1}\}$.

Hence, each i_n collects all the first n possible inputs for the Turing machine T. The arrow between i_n and i_{n+1} is the inclusion map between them.

The functor F applies one step s of the Turing machine to the chain $i_0(0) \hookrightarrow i_0(1) \hookrightarrow i_0(2) \cdots i_0(n) \hookrightarrow \cdots$ having a new chain $s(i_0(0)) \hookrightarrow s(i_0(1)) \hookrightarrow s(i_0(2)) \cdots s(i_0(n)) \hookrightarrow \cdots$, i.e., $F_0(1)$.



[1]Obs: $F_0 = i_0$

Considering $\omega = 0 \to 1 \to 2 \to \ldots \to n \to \ldots$, given a Turing machine T, the functor $F_T \in (FinSets^{\omega})^{\omega}$ is just as defined.

Let $\omega \times \omega$ be provided with the Grothendieck topology [2]. Then F_T satisfies the compatibility condition. So it is a sheaf.

We have, therefore, that:

- Every morphism $f : A \to B$ in $FinSets$ is Turing-computable. Even considering that $A, B \subseteq Data$ and $Data$ is r.e. and non-recursive;
- Every Turing-computation can be represented in $FinSets^{\omega}$;
- Every Turing-computable, possibly, partial function can be represented by a sheaf on the site $\langle sieves(\omega), FinSets^{\omega} \rangle$.

The objects of $(FinSets^{\omega})^{\omega}$ of the form F_T for some Turing machine T form a subcategory **Tur**.

The main contribution of this work, is to show that every computational sheaf C is essentially F_T for some Turing Machine T, so **Tur** is a reflective subcategory of $(FinSets^{\omega})^{\omega}$. With this, we conclude that we may have an enlightful geometrical model for computable functions that can be extended to be a refined model to provide meaningful semantics for non-terminating Turing machines.

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A Tableau for Ecumenical Propositional Logic

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Abstract

In [2], J.-Y. Girard asks whether is it possible to handle the well-known distinction between "classical" and "intuitionistic" not through a change of system, but through "a change of formulas." While considering different but related thoughts, Krauss figured that, with the adequated ground, such a system would offer a "constructively valid refinement of classical reasoning." [3] Interestingly enough, this pluralist glimpse of paradise would be feasible, according to him, with a simple notational movement "(...) to distinguish between two different kinds of logical operators requires some additional effort. However, this effort is only notational." [5]

It was in that spirit that ecumenical logic flourished – as a systematic way to hold together these two logics which are, in many senses, incompatible. This apparent controversial scenario vanishes with a deal: in Prawitz's ecumenical system [6], if classical and intuitionistic logicians agree to share the semantics of conjunction, negation, and bottom, they could define the semantics of their respective disjunction ($\vee_c, \vee_i$) and implication ($\to_c, \to_i$) at the same level. In this mixed logic, the intuitionistic logician is happy to consent with the classical logician that $A \vee_c \neg A$ is trivially valid while the classical logician has no reasons to disagree about the non-validity of $A \vee_i \neg A$: after all, it is not always true that one has a proof of A or a proof of $\neg A$.

Recently, those operators received a Kripke semantics formalization [1]. Following works of Pereira and al., a natural deduction system (NEp) [1] and a sequent calculus ($LEci$) was presented [8]. Ecumenical modalities were also being studied [7]. In the wake of these works, we propose a tableau system for ecumenical propositional logic and prove its soundness and completeness.

In our system, we define special rules for F signed negation, intuitionistic implication, and intuitionistic disjunction [4]. We have also provided an implementation of the system on Coq proof assistant, which can be used to automate ecumenical deductive reasoning.

$$\frac{S, \langle F(\varphi \to_i \psi), k_n\rangle}{S_T, \langle T\varphi, k_{n+1}\rangle, \langle F\psi, k_{n+1}\rangle} \qquad \frac{S, \langle F\left(\varphi \vee_i \psi\right), k_n\rangle}{S, \langle F\varphi, k_n\rangle \parallel S, \langle F\psi, k_n\rangle} \qquad \frac{S, \langle F\left(\neg\psi\right), k_n\rangle}{S_T, \langle T\psi, k_{n+1}\rangle}$$

$$\frac{S, \langle Tp, i\rangle, \langle L, j\rangle}{S, \langle Tp, j\rangle} i \neq j$$

The nodes of the tree are ordered pairs with the following structure: $\langle L, k\rangle$, where L is a signed ecumenical proposition and k an index that is globally new whenever a node is generated by $F\neg A$ or $FA \to_i B$ rules. To prove completeness, we add an extra rule which pushes forward every atomic validity whenever fresh indexes are created on the branch.

We had also obtained a Hilbert calculus for propositional ecumenical logic, which we had proved to be equivalent to the NEp natural deduction system on Coq proof assistant. Furthermore, we formalize, using Coq, the proof of NEp soundness. In future works, we plan to expand our tableau to predicates as well as to include other ecumenical systems in our investigations.

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Combining belief, knowledge and evidence

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Abstract

In 1933, Gödel [2] announced his conjecture that prefixing the necessity symbol $\Box$ (box) to each subformula of Intuitionistic Propositional Logic (IPC) results in a sound and faithful translation of IPC into the classical modal logic $S4$, i.e. the following holds for any formula φ of IPC:

$$\vdash_{IPC} \varphi \Longleftrightarrow \vdash_{S4} \varphi^*,$$

where φ^* is the result of Gödel's translation. This conjecture was proved a few years later by McKinsey and Tarski [3]. Intuitionistic semantics can be informally described by the Brouwer-Heyting-Kolmogorov (BHK) interpretation where intuitionistic truth is generally understood as "proof" in some intuitive, constructive sense. So in view of the above result which interprets IPC in $S4$, Gödel considered classical modal system $S4$ as a (classical) provability calculus.

However, under Gödel's translation (or related translations of IPC into $S4$ found in the literature), $S4$ contains IPC only in some codified form. In particular, the necessity symbol cannot be regarded as a direct proof predicate (a predicate of intuitionistic truth) in the following sense: $\Box\varphi$ is classically true iff φ is intuitionistically true.

A direct embedding $\varphi \mapsto \Box\varphi$ of IPC into a hierarchy of classical modal systems $L \subset L3 \subset L4 \subset L5$ was established by Lewitzka [4]. Besides the disjunction principle $\Box(\varphi \vee \psi) \rightarrow \Box\varphi \vee \Box\psi$, those modal systems contain corresponding axioms of Lewis modal systems $S1 \subset S3 \subset S4 \subset S5$, respectively. For any formula φ of IPC, it holds that

$$\vdash_{IPC} \varphi \Longleftrightarrow \vdash_{\mathcal{L}} \Box\varphi,$$

where $\mathcal{L} \in \{L, L3, L4, L5\}$. Moreover, for any formula ψ of the extended modal language and any given $\mathcal{L}$-model: $\Box\psi$ is true iff ψ is intuitionistically true. Thus, $\Box$ is a predicate for intuitionistic truth. The term 'intuitionistically true' refers here to the usual BHK interpretation augmented with the following clause: a proof of $\Box\varphi$ consists in presenting an *actual proof* (i.e. an available, effected construction) of φ. The classical reading of $\Box\varphi$ is: "φ has an actual proof". As argued in [5, 6], within the hierarchy $L \subset L3 \subset L4 \subset L5$, modal logic $L5$ should be regarded as the adequate system for classical reasoning about intuitionistic truth. In fact, the $S5$-principle $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$, contained in $L5$, is sound w.r.t. the extended BHK semantics. Moreover, arguing in a more formal way, $L5$ is sound and complete with respect to a relational semantics based on intuitionistic general frames [5] and thus is in accordance with intuitionistic reasoning. If the modal axioms of $S5$ (augmented with the disjunction principle) are adequate for the reasoning about *proof*, i.e. constructive truth, then the question arises whether in modal logic $S5$ itself the term 'necessity' can be re-interpreted as a non-constructive counterpart of *proof*. We propose here the intuitive concept of 'evidence' or 'certainty'. A formula $\Box\varphi$ then reads as "truth of φ is evident" in the sense that truth of φ is immediately given, i.e. immediately accessible/comprehensible/recognizable such that no effort nor resources must be spent. In a

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similar way as an *actual proof* is considered in [5, 6], we regard 'evidence' or 'certainty' as an absolute and static concept:

"A proposition is evident or its evidence is impossible."

This is in contrast to the more general concept of 'truth': truth of a proposition might be possible even if the proposition is not true in the given situation (world). The assumption above justifies the validity of the $S5$-principle $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (note that $\Box\neg\Box\varphi$ is equivalent to $\neg\Diamond\Box\varphi$, i.e. "$\Box\varphi$ is impossible".) The concept of 'evidence' ('certainty') gives rise to an epistemic approach. We assume the existence of an agent that is able to believe in propositions and even to discover the truth of propositions if this is not too difficult. Of course, the agent believes and knows a proposition φ whenever φ is evident. If there is no evidence of φ, the agent may achieve belief or knowledge of φ by investing some effort (of course, knowledge can be achieved only if the proposition is true). To be more precise, we must distinguish between formulas and their denotations: a formula *denotes* a proposition which is given as an element of a model-theoretic propositional universe (a Boolean algebra). The set of true propositions, modeled by an ultrafilter, as well as the whole universe of propositions, is partially ordered by strict implication, i.e. by the ordering of the given Boolean lattice: $\Box(\varphi \rightarrow \psi)$ means that the proposition (denoted by) φ is (in some not further specified, intuitive sense) more difficult than proposition ψ, or "belief/knowledge of φ is harder to obtain than belief/knowledge of ψ". Consequently, our approach combines modal system $S5$ with epistemic operators and principles for belief and knowledge. In contrast to the usual possible worlds semantics, we model classical truth, belief and knowledge as appropriate filters of a given Boolean algebra. We claim here that our algebraic semantics has a relational counterpart (based on $S5$-models) in a similar way as algebraic semantics of epistemic extensions of $L5$ have relational counterparts based on intuitionistic general frames (cf. [5]). However, belief and knowledge in such relational models are represented directly by sets of sets of possible worlds (i.e. sets of propositions) instead of being modeled by accessibility relations. So from a technical point of view, our relational models would represent an alternative semantics to the traditional possible worlds approach to belief and knowledge. Finally, we discuss possible extensions of our axiomatic system by further epistemic principles and compare these with other systems that combine belief and knowledge found in the literature (cf. [1, 7, 8]).

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Quase-Nelson: lógica e fragmentos

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A lógica construtiva com negação forte (**N3**) foi introduzida por David Nelson em [6], no qual o conectivo unário de negação $\sim$ é involutivo. Todavia, a lógica paraconsistente de Nelson (**N4**), uma generalização de **N3** obtida ao abandonar o axioma da explosão $p \to (\sim p \to q)$, aparece mais tarde numa publicação junto com A. Almukdad [1]. É sabido que **N3** e **N4** são lógicas não-clássicas que possuem, como contrapartidas algébricas, a variedade das álgebras de Nelson e a variedade dos N4-reticulados, respectivamente. Uma outra generalização de **N3** é obtida ao eliminar a lei da dupla negação $\sim\sim p \to p$, isto é: a lógica quase-Nelson (**QNL**), que foi introduzida em [9] e cuja contrapartida algébrica é a variedade das álgebras quase-Nelson.

U. Rivieccio [7] introduziu a classe dos quase-N4-reticulados (QN4-reticulados), como uma generalização comum das variedades dos N4-reticulados e das variedades das álgebras quase-Nelson. Noutras palavras, os N4-reticulados acabam sendo precisamente os quase-N4-reticulados satisfazendo a lei da dupla negação, e as álgebras quase-Nelson são precisamente os QN4-reticulados que satisfazem a lei explosiva. As álgebras de Nelson, os N4-reticulados e as álgebras quase-Nelson podem ser representados através de estruturas *twist*. Para realizar isso, esta representação emprega estruturas *twist* definidas sobre álgebras Brouwerianas[1] enriquecidas com um operador de núcleo.

Dada uma álgebra $\mathbf{A}$ tendo uma operação $\to$ e os elementos $a, b \in A$, definimos as relações $\equiv$ e $\preceq$ como segue: $a \preceq b$ se, e somente se, $a \to b = (a \to b) \to (a \to b)$, e $\equiv := \preceq \cap (\preceq)^{-1}$. Assim, temos $a \equiv b$ se, e somente se, $a \preceq b$ e $b \preceq a$. Diante do exposto, temos condições de definir QN4-reticulados.

Definição 1 ([7], Def. 3.2)**.** Uma *quase-N4-reticulado (QN4-reticulado)* é uma álgebra $\mathbf{A} = \langle A; \wedge, \vee, \to, \sim \rangle$ do tipo $\langle 2, 2, 2, 1 \rangle$ satisfazendo as seguintes propriedades:

(QN4a) O reduto $\langle A; \wedge, \vee \rangle$ é um reticulado distributivo com ordem do reticulado $\leq$.

(QN4b) A relação $\equiv$ é uma congruência sobre o reduto $\langle A; \wedge, \vee, \to \rangle$ e o quociente $B(\mathbf{A}) = \langle A; \wedge, \vee, \to \rangle / \equiv$ é uma álgebra Brouweriana. Além disso, o operador $\Box$ dado por $\Box[a] := \sim\sim a / \equiv$ para todo $a \in A$ é um núcleo, então a álgebra $\langle B(\mathbf{A}), \Box \rangle$é uma álgebra Brouweriana nuclear.

(QN4c) Para todos $a, b \in A$, é válido que $a \leq b$ se, e somente se, $a \preceq b$ e $\sim b \preceq \sim a$.

(QN4d) Para todos $a, b \in A$, é válido que $\sim(a \to b) \equiv \sim\sim(a \wedge \sim b)$.

(QN4e) Para todos $a, b \in A$,

(QN4e.1) $a \leq \sim\sim a$.

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[1] Uma *álgebra Brouweriana* é precisamente o sub-reduto livre do 0 de uma álgebra de Heyting.

(QN4e.2) $\sim a = \sim\sim\sim a$.

(QN4e.3) $\sim(a \vee b) = \sim a \wedge \sim b$.

(QN4e.4) $\sim\sim a \wedge \sim\sim b = \sim\sim(a \wedge b)$.

A contrapartida lógica dos QN4-reticulados ($\mathbf{L}_{\mathrm{QN4}}$) foi introduzida em [4] através de um cálculo estilo Hilbert. O cálculo para $\mathbf{L}_{\mathrm{QN4}}$ consiste nos seguintes esquemas de axiomas junto com a única regra de inferência MP (*modus ponens*): $p,\, p \to q \vdash q$.

Ax1 $p \to (q \to p)$

Ax2 $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

Ax3 $(p \wedge q) \to p$

Ax4 $(p \wedge q) \to q$

Ax5 $(p \to q) \to ((p \to r) \to (p \to (q \wedge r)))$

Ax6 $p \to (p \vee q)$

Ax7 $q \to (p \vee q)$

Ax8 $(p \to r) \to ((q \to r) \to ((p \vee q) \to r))$

Ax9 $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$

Ax10 $\sim(p \to q) \leftrightarrow \sim\sim(p \wedge \sim q)$

Ax11 $\sim(p \wedge (q \wedge r)) \leftrightarrow \sim((p \wedge q) \wedge r)$

Ax12 $\sim(p \wedge (q \vee r)) \leftrightarrow \sim((p \wedge q) \vee (p \wedge r))$

Ax13 $\sim(p \vee (q \wedge r)) \leftrightarrow \sim((p \vee q) \wedge (p \vee r))$

Ax14 $\sim\sim(p \wedge q) \leftrightarrow (\sim\sim p \wedge \sim\sim q)$

Ax15 $p \to \sim\sim p$

Ax16 $p \to (\sim p \to \sim(p \to p))$

Ax17 $(p \to q) \to (\sim\sim p \to \sim\sim q)$

Ax18 $\sim p \to \sim(p \wedge q)$

Ax19 $\sim(p \wedge q) \to \sim(q \wedge p)$

Ax20 $(\sim p \to \sim q) \to (\sim(p \wedge q) \to \sim q)$

Ax21 $(\sim p \to \sim q) \to ((\sim r \to \sim s) \to (\sim(p \wedge r) \to \sim(q \wedge s)))$

Ax22 $\sim\sim\sim p \to \sim p$.

$\mathbf{L}_{\mathrm{QN4}}$ desfruta do Clássico Teorema da Dedução: $\Gamma, \alpha \vdash \beta$ é equivalente a $\Gamma \vdash \alpha \to \beta$. Para uma lógica algebrizável $\mathbf{L}$ [3, Def. 3.11], dizemos que $\mathbf{L}$ é *finitamente algebrizável* quando o conjunto de fórmulas de equivalências é finito, e dizemos que $\mathbf{L}$ é *BP-algebrizável* quando $\mathbf{L}$ é finitamente algebrizável e o conjunto de identidades definidoras é finito. Usando as seguintes abreviações:

$$x \Rightarrow y := (x \to y) \wedge (\sim y \to \sim x) \qquad x \Leftrightarrow y := (x \Rightarrow y) \wedge (y \Rightarrow x)$$

inferimos que $\mathbf{L}_{\mathrm{QN4}}$ é BP-algebrizável com a identidade definidora $E(\alpha) := \alpha \approx \alpha \to \alpha$ e a fórmula de equivalência $\Delta(\alpha, \beta) := \alpha \Leftrightarrow \beta$. Com base nesse resultado, obtemos uma axiomatização da semântica quase-variedade equivalente $\mathrm{Alg}^*(\mathbf{L}_{\mathrm{QN4}})$ de $\mathbf{L}_{\mathrm{QN4}}$. Como mostrado em [4, Cor. 1], a classe de álgebras introduzidas coincide com a variedade de QN4-reticulados, isto é, $\mathrm{Alg}^*(\mathbf{L}_{\mathrm{QN4}}) = \mathsf{QN4}$.

A lógica quase-Nelson (**QNL**), vista como uma lógica subestrutural, é a extensão axiomática do cálculo *Full Lambek* com as regras *exchange* e *weakening* ($\mathbf{FL}_{ew}$) pelo axioma de Nelson, a saber:

$$((p \Rightarrow (p \Rightarrow q)) \wedge ({\sim} q \Rightarrow ({\sim} q \Rightarrow {\sim} p)) \Rightarrow (p \Rightarrow q)$$

e tendo como contrapartida algébrica a variedade dos reticulados residuados chamada *álgebras quase-Nelson* (QNA). T. Nascimento e U. Rivieccio [5] iniciaram o estudo dos fragmentos de **QNL**, que correspondem aos sub-redutos de QNA. No artigo em questão, temos uma axiomatização do fragmento $\{\sim, \to\}$ (apelidado de *álgebras de implicação quase-Nelson*, QNI), para o qual foi introduzido um cálculo estilo Hilbert, que por sua vez é BP-algebrizável com respeito à variedade QNI. Dando continuidade aos estudos dos fragmentos de **QNL**, os autores detectaram que alguns deles não são algebrizáveis, no sentido de [2]; como exemplo deste caso, temos o fragmento $\{\sim, *\}$ cuja classe de sub-redutos de QNA é chamada monóides quase-Nelson. Entretanto, ainda estamos trabalhando na axiomatização dos fragmentos algebrizáveis, tais como $\{\sim, \to, *\}$ e $\{\sim, \to, \wedge\}$, cujo sub-redutos são denominados *quase-Nelson pocrims* e *quase-Nelson semihoops*, respectivamente. Convém destacar que a metodologia para caracterizar algebricamente tais fragmentos têm sido com a utilização da generalização das estruturas *twist* para as lógicas de Nelson e quase-Nelson (ver [8]).

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O Mapa de Abel-Jacobi em Jogos de Comparação de Modelos Finitos

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Resumo

A noção de jogos em teoria dos modelos finitos fornece uma maneira de provar os limites do poder expressivo de lógicas. Um tipo de jogo bastante comum e útil usado em aplicações da teoria dos modelos finitos à complexidade computacional descritiva é o que compara modelos (finitos). Grosso modo, os jogos de comparação de modelos são aqueles formados por dois jogadores, comumente chamados de *spoiler* e duplicador, que usam como "tabuleiro" duas estruturas $\mathfrak{A}$ e $\mathfrak{B}$, onde a finalidade do jogo é estabelecer que $\mathfrak{A}$ e $\mathfrak{B}$ não podem ser distinguidas em alguma lógica dada. Assim, o *spoiler* tenta provar que $\mathfrak{A}$ e $\mathfrak{B}$ são diferentes, enquanto que o duplicador tenta mostrar que $\mathfrak{A}$ e $\mathfrak{B}$ são "iguais" (isomórficas). É dito que $\mathfrak{A}$ e $\mathfrak{B}$ são indistinguíveis - de acordo com as regras do jogo - se o Duplicador tiver uma *estratégia vencedora*, de maneira que se $\mathfrak{A}$ e $\mathfrak{B}$ são isomórficas, então o Duplicador tem necessariamente uma estratégia vencedora. Alguns exemplos de jogos de comparação de modelos são os seguintes: jogos de Ehrenfeucht–Fraïssé; jogos de seixos; jogos de contagem; jogos de bijeção; jogos de partição; e jogos de mapeamentos inversíveis. Tais jogos são usados pra estabelecer resultados de inexpressabilidade: algo crucial para a separação de classes de complexidade via separação de lógicas.

Entretanto, as provas de inexpressabilidade baseadas em jogos frequentemente envolvem um intrincado argumento combinatório, tal que mesmo os resultados mais simples muitas vezes requerem argumentos intrincados. Por essa razão, foi sugerido por Fagin, Stockmeyer e Vardi [2] a construção de uma livraria de estratégias vencedoras para esses jogos. Em geral, o ideal seria termos uma coleção de ferramentas versáteis e de fácil aplicação para provar limites de expressabilidade. O objetivo deste trabalho é sugerir essas ferramentas.

O trabalho de Toshikazu Sunada [3–5] em cristalografia topológica possui como uma de suas engrenagens o conceito de *cobertura abeliana máxima* de um grafo. A cobertura universal de um grafo conexo X tem o grupo fundamental $\pi_1(X)$ como seu grupo de transformações de deck. Já a cobertura abeliana máxima, $\overline{X}$, tem a abelianização de $\pi_1(X)$ como seu grupo de transformações de deck. Assim, cobre todas as outras coberturas conexas de X cujo grupo de transformações de deck é abeliano. Isso mostra a existência de uma conexão bastante estreita entre a cobertura abeliana máxima e homologia, tendo em vista que a abelianização de $\pi_1(X)$ é o primeiro grupo de homologia $H_1(X, \mathbb{Z})$.

Agora, o mapa de Abel-Jacobi clássico é um mapeamento $J : S \to \mathbb{C}^g/\Lambda$, onde S é uma curva algébrica (superfície de Riemann de genus g) e $\Lambda \subset \mathbb{C}^g$ é o reticulado de períodos. Ou seja, existem g diferenciais holomórficos linearmente independentes $\omega_1, ..., \omega_g$ em S, e se $\{c_j\}_{j=1}^{2g} \subset H_1(S, \mathbb{Z})$ é uma coleção de ciclos básicos, então os vetores $v_j = \langle c_j, \omega \rangle = (\int_{c_j} \omega_1, ..., \int_{c_j} \omega_g)$ formam uma base de um reticulado Λ. Com esses dados, o mapa de Abel-Jacobi $J : S \to \mathbb{C}^g/\Lambda$ é dado por $J(p) = (\int_{p_0}^{p} \omega_1, ..., \int_{p_0}^{p} \omega_g) \bmod \Lambda$, sendo um isomorfismo quando a curva S é elíptica.

O mapa de Abel-Jacobi, em sua forma discreta, tem sido utilizado na construção de realizações padrão de coberturas abelianas máximas de grafos em cristalografia topológica (ver [1,3–5]). O presente trabalho busca, então, construir mapas discretos de Abel-Jacobi para jogos, por meio da descrição de cociclos simpliciais explícitos $\omega_i (1 \leq i \leq k)$, nas estruturas que compõem o "tabuleiro", onde tais cociclos serão os análogos discretos dos diferenciais holomórficos descritos acima. Isso fornecerá as ferramentas necessárias para comparar o que vou chamar de "assinatura combinatória" de jogos, vistos como relações entre complexos. Eu irei fornecer um exemplo concreto do tipo de construção que sugiro aqui, a fim de demonstrar a viabilidade dessa nova maneira de abordar jogos de comparação de modelos.

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Towards Modular Mathematics

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Abstract

Synthetic reasoning is a powerful method, albeit circumscribed, of exploring mathematical landscapes. A synthetic researcher works by analyzing current mathematical efforts in an area of interest, extracting distinctive features from such area's reasoning, and *synthetizes* those features into an axiomatic system that allows (and makes convenient) exactly *that* kind of reasoning. One example is well-known: Kock's Synthetic Differential Geometry (SDG) [1], and related works. But there are others: Bauer's Synthetic Computability Theory [2]; Synthetic Probability and Statistics [3]; and even Kock's alternative theory for SDG [4].

Synthetic theories often follow a pattern: they are type theories intended to be interpreted inside sufficiently structured categories. This makes its power doublefold: on one side, that axioms better reflect mathematicians' internal assumptions; on the other, the fact that that theory (and its theorems) can be interpreted inside other mathematical worlds.

For example: in SDG, one postulates a type R with the structure of a ring, defines a subset $D = \{d : R \mid d^2 = 0\}$ using the underlying logic's tools, and postulates the following axiom

$$\forall f : D \to R \, \exists b : R \, \forall d : D \, . \, f(d) = f(0) + b \cdot d$$

Here, we are expressing the axiom in some kind of type theory, and it can be interpreted in a category so that D and R are objects and f, b and d are arrows (provided that category has, *e.g.*, products and pull-backs). Arguably, such an axiom captures the structural meaning of differentiation (see [1] and [4] for a discussion).

But powerful as it may be, synthetic reasoning only goes so far. Theory-crafting is a meticulous and artisanal job, which is what allows it to be so well-fitted to a particular domain. But, without the syntactical tools to do otherwise, a theory-crafter is bound to make their theories restricted to that domain, and unable to communicate with others but for more handwork. However, a syntactical "meta-framework", so to say, that incorporates lessons from Universal Logic, would aid the theorycrafter in connecting theories together.

Universal Logic (UL) has been an area of active study for the last few decades (see, *e.g.*, [5] or [6]). One might summarize its goals in three separate points: how to *identify*, *translate*, and *combine* logics? We should then require a complete formalism from UL to allow its users to specify logical theories (whatever the system commits to as "a logic", part of the first question) that could then be compared, translated, and combined with each other in mechanical ways. That is analogous to how category theory structures and guides mathematical enquiry: there is still work to be done, but the framework guides the mathematician's efforts in the right direction (or more so than when not using it).

A UL formalism, then, would be a suitable workbench for a theory-crafter. Everything in math is a model of a theory – indeed, that is just a consequence of accepting synthetic reasoning: if a theory provides vocabulary and rules for reasoning about some objects, those objects must, individually or collectively, be models for that theory (or it'd be a bad theory). Examples are abundand: plain algebraic objects are interpretations of their first order theories into some kind of set theory; SDG shows the same for geometric objects. In a informal sense, every definition creates this sort of theory-model relation.

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There is also a well-known cartesian structure to model-taking and theory-combining. For example, in categorial logic there are synthetic objects called "sketches" that admit models in some categories. Given two sketches $\mathcal{S}$ and $\mathcal{T}$, a sufficiently equiped category C and suitable definitions of $\otimes$ and Mod, it can be shown that

$$\mathrm{Mod}(\mathcal{S} \otimes \mathcal{T}, C) \simeq \mathrm{Mod}(\mathcal{S}, \mathrm{Mod}(\mathcal{T}, C))$$

Or, as is didactically explained in a talk by Maaike Zwart [7], under the formalization of algebraic and composite theories, the theory of rings is given as the composite theory of monoids and then abelian groups (but not the other way around!). This shows a formal instance *in the wild* that perhaps confirms, perhaps corrects, mathematicians' intuition.

So a theory-crafter could combine synthetic theories – which, at least in principle, can encompass all of mathematics – to obtain new mathematical notions; perhaps even old ones, but decomposed in novel ways. These combinations should behave well, and commute with the taking-of-models, as in the above expression. A combined theory can reveal interesting interactions that are present in the relevant objects. One example of a possible combination: Lie groups are nothing more than models of a theory of a group in the world of differential manifolds; or alternatively, a differential manifold in the world of groups; or, even, a model of a combined theory of groups and manifolds. The point is that a UL formalism should strive to capture those practical notions of theory-combination found in the wild.

One candidate for such a formalism is MMT, a system proposed with the aim of providing a "module system for mathematical theories" [8, p. 3]. In MMT, "theories" are sequences of declarations that may depend on previous declarations. There is a conflation of types *qua* types and propositions; and between terms, signature members, axioms, and proofs. A theory of monoids could look like this:

$$\begin{aligned}
M &: \mathrm{Type} \\
\cdot &: M \to M \to M \\
e &: M \\
\mathsf{assoc} &: (x, y, z : M) \to x \cdot (y \cdot z) = (x \cdot y) \cdot z \\
\mathsf{unit}_{\mathrm{L}} &: (x : M) \to x = e \cdot x \\
\mathsf{unit}_{\mathrm{R}} &: x \cdot e = x
\end{aligned}$$

Morphisms work purely syntactically, taking declared names to complex terms. An interpretation of the theory of a monoid as the monoid of endofunctions in some sort of set theory could look like this:

$$\begin{aligned}
M &\mapsto X^X && (X \text{ a previously constructed set}) \\
\cdot &\mapsto \lambda f \lambda g(\lambda x \,.\, f(g(x))) \\
e &\mapsto \lambda x \,.\, x \\
\mathsf{assoc} &\mapsto [\dots] \text{ (proof of associativity)} \\
\mathsf{unit}_{\mathrm{L}} &\mapsto [\dots] \text{ (proof of left unitality)} \\
\mathsf{unit}_{\mathrm{R}} &\mapsto [\dots] \text{ (proof of right unitality)}
\end{aligned}$$

MMT seems precisely suitable to the definition, combination and translation of mathematical theories[1]. Given the proper tools to quickly construct and transform synthetic theories, it could become a powerful tool for modular mathematics.

However, there are some limitations for such a system – or perhaps some underdevelopment. A proper UL formalism must provide, besides rules and syntax, a toolbox for creating and transforming theories, and a knowledge-base of examples of applications of that toolbox to standard mathematical problems, and perhaps a demonstration of applications to new ones. In

[1]Indeed, it *was* designed for that, afterall. See the titles of Rabe's papers.

addition, a closed monoidal structure is missing; that would allow quick and mechanical taking of models and combination of theories.

At last, we might punctuate that a formalism for UL allied with the perspective of synthetic reasoning can allow logic to become an engine for mathematical praxis: the practicing mathematician may analyze his own practice, and synthetize new ways of doing it, which will fuel more analyzis (as exemplified by the evolution from [1] to [4]).

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Exploring two completeness conditions on quantale valued sets

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Abstract

In the 1970s, the topos of sheaves over a locale/complete Heyting algebra $\mathbb{H}$, denoted as $Sh(\mathbb{H})$, was described, alternatively, as a category of $\mathbb{H}$-sets [3]. More precisely, in [6], there were three categories whose objects were locale valued sets that are equivalent to the category of sheaves over a locale $\mathbb{H}$. Two different notions of separability and completeness have been proposed. On the one hand, the traditional notions of these properties in $Sh(\mathbb{H})$ can be translated to appropriate definitions in $\mathbb{H}$-sets. In addition, Scott's notion of singletons, a definition that is inspired from the ordinary singleton set, leads alternative notions of completeness and separability.

Later, more general categories have been proposed, replacing locales by the Mulvey's quantales [1], as studied in [4]. Instead considering the traditional idempotent non-commutative quantales, that arose from certain C^*-algebras and its relationships with quantum physics; we are following proposals like in [2] and [5], that have connections with affine, fuzzy, and continuous logic. In this work we consider a class of commutative and integral/semicartesian quantales, which includes both the quantales of the ideals of commutative unital rings, MV-Algebras, Heyting Algebras and $([0,1], \leq, \cdot)$ – which is isomorphic to $([0,\infty], \geq, +)$.

def.: A commutative semicartesian quantale $(\mathbb{Q}, \odot, 1, \leq)$ is:

(i) $(\mathbb{Q}, \leq)$ is a complete lattice;

(ii) $(\mathbb{Q}, \odot, 1)$ is a commutative monoid where $1 = \top$ (integral/ semicartesian);

(iii) the general distributive law $a \odot \bigvee_{i\in I} b_i = \bigvee_{i\in I} a \odot b_i$ holds.

There is an important technical "strength" condition considered by Höhle in [2]. With this, Scott-completeness makes sense for commutative semicartesian quantales. $\mathbb{Q}$ is "strong" if:

(iv) if $e \odot e = e$ and $e \leq \bigvee A$, then $e \leq \bigvee\{a \odot a : a \in A\}$.

The main examples of strong quantales are Heyting Algebras and $([0,1], \leq, \cdot)$. Some MV-Algebras, like the Chang's $([0,1], \wedge, \vee, \oplus, \odot, 0, 1)$ [8], are not strong.

def.: Let $\mathbb{Q}$ be a commutative integral quantale. A $\mathbb{Q}$-set is a pair (X, δ) where X is a set and $\delta : X \times X \to \mathbb{Q}$ is a function satisfying:

$$\delta(x,y) = \delta(y,x)$$
$$\delta(x,y) \odot \delta(y,z) \leq \delta(x,z)$$
$$\delta(x,y) \odot \delta(y,y) = \delta(x,y)$$

We abbreviate $\delta(x,x)$ by Ex, read as the "extend" of x. It is immediate that Ex is idempotent. If $\mathbb{Q}$ is a Heyting algebra $\mathbb{H}$, then $\odot = \wedge$ and the last condition is automatically satisfied. When $\mathbb{Q} = ([0,\infty], \geq, +)$, the category of $\mathbb{Q}$-Sets is equivalent to the extended pseudometric spaces, and if these sets are complete, then it is equivalent to the category of extended metric spaces.

There are at least two different notions of morphisms for $\mathbb{Q}$-sets $(X, \delta) \to (Y, \delta')$: one is "relational" in nature, the other is functional.

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def.: A *relational* morphism of $\mathbb{Q}$-sets $\varphi : (X, \delta) \to (Y, \delta')$ is a function $\varphi : X \times Y \to \mathbb{Q}$ such that:

$$\varphi(x,y) \odot \delta(x,x') \leq \varphi(x',y)$$
$$\varphi(x,y) \odot \delta'(y,y') \leq \varphi(x,y')$$
$$\varphi(x,y) \odot \varphi(x,y') \leq \delta'(y,y')$$
$$\varphi(x,y) \odot Ex \odot E'y = \varphi(x,y)$$
$$\bigvee_{y \in Y} \varphi(x,y) = Ex$$

This class of objects and morphisms with obvious composition defines a category whenever the $\mathbb{Q}$ is strong. It is not known whether this condition is strictly necessary.

def.: A *functional* morphism of $\mathbb{Q}$-sets $f : (X, \delta) \to (X', \delta')$, on the other hand, is a function $f : X \to X'$ such that:

$$\delta(x,x) = \delta'(f(x), f(x))$$
$$\delta(x,y) \leq \delta'(f(x), f(y))$$

Here too there is an obvious categorical structure for those morphisms given any semicartesian quantale $\mathbb{Q}$.

def.: A singleton of (X, δ) is a function $\sigma : X \to \mathbb{Q}$ such that:

$$\sigma(x) \odot Ex = \sigma(x)$$
$$\sigma(x) \odot \delta(x,y) \leq \sigma(y)$$
$$\sigma(x) \odot \sigma(y) \leq \delta(x,y)$$

Additionally, a singleton is strict iff:

$$\sigma(x) \odot \bigvee_{x' \in X} \sigma(x') = \sigma(x)$$

And it is representable iff:

$$\exists x_\sigma \in X (\forall y \in X (\sigma(x) = \delta(x_\sigma, y))$$

def.: A $\mathbb{Q}$-Set X is separable or extensional iff any of these equivalent conditions hold:

(i) $Ex = \delta(x,y) = Ey \Rightarrow x = y$

(ii) $Ex \vee Ey = \delta(x,y) \Rightarrow x = y$

(iii) $\forall z \in X (\delta(x,z) = \delta(y,z)) \Rightarrow x = y$

def.: A subset $A \subseteq X$ is compatible iff:

$$\forall a, b \in A (Ea \odot Eb = \delta(a,b))$$

def.: We call X gluing-complete iff for all A compatible exists x_A such that:

(i) $\forall a \in A (\delta(x_A, a) = Ea)$

(ii) $Ex_A = \bigvee_{a \in A} Ea$

def.: Let $\mathbf{s}(X)$ the set of strict singletons of X. We call X Scott-separable if, and only if, the function $\eta : X \to \mathbf{s}(X), \eta(x) = \sigma_x : X \to \mathbb{Q}, \sigma_x(y) = \delta(x,y)$ is injective. In addition, X Scott-complete if, and only if, η is a bijection.

Despite some folkloric misconceptions about these notions, a simple counter-example (in a finite boolean algebra) shows that these two definitions of completeness are not equivalent.

The goal of the present work is to examine these two notions of completeness (and separability): (i) via (unique) gluing of compatible families (gluing-complete $\mathbb{Q}$-sets), (ii) via (unique) representability of strict singletons (Scott-complete $\mathbb{Q}$-sets). We have shown that:

1. Every Scott-complete $\mathbb{Q}$-set is gluing-complete;
2. Both full subcategories of gluing-complete and Scott-complete $\mathbb{Q}$-sets with functional morphisms are reflective;
3. For "strong" quantales [2], the categories of Scott-complete $\mathbb{Q}$-sets and relational morphisms and Scott complete $\mathbb{Q}$-sets functional morphisms are isomorphic.

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Valuation semantics and modal logics

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Abstract

This work discusses the application, to several systems of normal and non-normal modal logic, of the valuation semantics technique proposed by A. M. Loparić for the basic normal modal logic **K**. We review results already obtained, showing two other ways of defining valuations for a modal logic, and also present some new results about classical modal logics, as well as a solution to a problem left open in previous works, that of provinding a valuatoin semantics for **S4**.

A *valuation* for a logic L is a function from the set $\mathcal{F}_L$ of all formulas of L to the set $\{1, 0\}$ of truth-values satisfying certain conditions (which vary depending on L). For classical propositional logic **PL**, for example, a valuation v is a function from $\mathcal{F}_{\mathbf{PL}}$ to the set $\{1, 0\}$ such that $v(\neg A) = 1$ iff $v(A) = 0$; $v(A \wedge B) = 1$ iff $v(A) = 1$ and $v(B) = 1$; and so on.

Valuation semantics were presented for many logics; among them we have, for instance, da Costa's paraconsistent logics $\mathbf{C}_n$ [1] and $\mathbf{C}_\omega$ [3], the modal logic **K** [2], some temporal and modal-temporal logics [5],[6], usual normal logics and some classical modal logics [7], Johansson's minimal logic and intuitionistic logic [4].

Valuations for **PL** can be easily defined because classical operators are truth-functional (and the same goes for other truth-functional many-valued logics). For modal logics, however, we need to add conditions specifying how to deal with modal operators; which conditions precisely will depend on the modal logic in question. In general, we would like to have something like the following:

- for any valuation v, if $v(\Box A) = 0$ then there is a valuation v' such that $v'(A) = 0$ and, for every formula $\Box B$ such that $v(\Box B) = 1$, $v'(B) = 1$;

and then we would also add some other clause dealing with the case in which $v(\Box A) = 1$. But evidently we cannot use this, on pain of circularity, to define a valuation. So we will need first to define certain functions and then define valuations in terms of them. This can be done in different ways, and we will show some examples. We can define valuations (i) directly in terms of, say, valuations for **PL** (what can be done for logics like **S0.5**, for instance, where the necessitation rule is restricted to tautologies), or (ii) we can use the modal degree of formulas, or (iii) we can use finite sequences of formulas closed under subformulas, as Loparić originally did for **K**, and we will illustrate this with our (Loparić's and mine) solution to **S4** (a problem left open in previous works), as well as pointing out how to modify the definitions to handle other systems.

For modal logics, valuation semantics allow us to dispense with the machinery of possible worlds and accessibility relations. Another advantage of the method of valuations is that usually we obtain a decision method for the logic question based on valuation tables.

We finish by mentioning some open problems; for example, how to valuations to normal modal logics characterized by adding axioms $\Diamond^k \Box^l A \to \Box^m \Diamond^n A$ to **K**.

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Três Particularidades de Semânticas Prova-teóricas: Restrições Estruturais, Domínios Extensionais e Bases Subestruturais

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Resumo

A abordagem das semânticas prova-teóricas pode ser definida como aquela na qual o conceito de prova, e não o de verdade, é usado como noção base da análise semântica. Diversas propostas já foram desenvolvidas nesta linha, que engloba desde definições em termos de reduções de argumentos [1] até definições que usam combinações de sistemas atômicos com clásulas semânticas [2]. Naturalmente, embora isto não seja uma característica essencial da abordagem, seu desenvolvimento é considerado especialmente relevante para o estudo de lógicas construtivas, tais como a lógica minimal e a intuicionista.

Em 2015, resultados de incompletude com relação à lógica intuicionista foram obtidos para uma noção promissora baseada em cláusulas semânticas e sistemas atômicos [3]. Apesar disso, foram obtidos resultados mostrando que mesmo neste contexto é possível evitar a incompletude caso sejam promovidas mudanças nas definições originais, tais como na cláusula da disjunção [4]. Em um artigo co-autorado atualmente em revisão, também conseguimos mostrar que é possível provar completude caso combinemos uma noção generalizada de validade prova-teórica com versões levemente modificadas das cláusulas originais.

Considerando que estas semânticas se apresentam como alternativas às semânticas verofuncionais, é natural que nos perguntemos quais diferenças podem existir entre propostas prova-teóricas e propostas tradicionais baseadas em modelos. Este será o tópico da apresentação, que tem por objetivo expor três particularidades técnicas (com interessantes repercussões filosóficas) de semânticas baseadas em sistemas atômicos que não parecem possuir paralelos nas semânticas de Kripke.

A primeira particularidade decorre diretamente da riqueza estrutural dos sistemas atômicos, que nos permite criar restrições à expansão da estrutura semântica utilizando a estrutura interna de um ponto inicial. Enquanto os "mundos" dos modelos de Kripke não possuem nenhuma "estrutura interna" e, portanto, nos permitem apenas especificar quais átomos valem e quais não valem em cada ponto, diferentes sistemas atômicos podem admitir diferentes provas dos mesmos átomos, o que permite a criação de estruturas semânticas mais expressivas.

Uma vez que a estrutura do sistema atômico base (que equivaleria a um "mundo inicial" da semântica de Kripke) pode impor restrições estruturais sobre propriedades de suas possíveis extensões (que equivaleria a uma restrição sobre quais mundos podem ser "acessíveis" ao mundo inicial), a adoção de sistemas atômicos efetivamente aumenta a expressividade dos modelos e permite a criação de estruturas semânticas consideravelmente mais ricas. A modificação nos permite, por exemplo, constatar que uma estrutura semântica valida a implicação $a \to b$ (com a e b atômico) simplesmente observando que o sistema atômico inicial contém regras que permitem a derivação de b a partir de a; ao revés, nos modelos de Kripke, é impossível (em regra) inferir a validade desta implicação em todo o modelo a partir de informações contida apenas no "mundo inicial". Em algumas noções prova-teóricas é possível até mesmo provar que modelos de Kripke correspondem a uma classe de estruturas semânticas com pouca complexidade interna, de modo que o enriquecimento semântico não parece ser acompanhado de nenhuma espécie de perda.

A segunda particularidade diz respeito a uma nova definição de domínios lógicos, possibilitada pela estrutura interna dos sistemas atômicos de primeira ordem. Enquanto nas semânticas de Kripke o domínio precisa ser especificado externamente, a nova semântica nos permite extrair domínios diretamente das regras contidas em um sistema atômico (contanto que algumas

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restrições específicas sejam usadas), fazendo com que a noção de domínio se torne puramente extensional. Isto permite uma nova validação do slogan "*meaning is use*", já que o domínio passa a ser determinado diretamente pelo uso dado a constantes lógicas pelas regras atômicas. Além disso, uma extensão das definições para a lógica de segunda ordem parece permitir a criação de uma nova semântica correta, completa e com justificação teórica robusta.

Do ponto de vista filosófico, esta característica parece permitir que a seleção de um domínio específico possa ser teoricamente justificada pelo tipo de regra utilizada em um sistema. Do ponto de vista técnico, esta característica facilita a análise semântica ao permitir que o lógico não se preocupe com a especificação de um domínio, já que este será extraído diretamente das regras do sistema. Não obstante, embora a definição de domínio passe a ser puramente extensional, algumas técnicas permitem seu inflacionamento artificial, o que torna possível uma recuperação da liberdade existente na "especificação externa" caso isto seja necessário para algum propósito.

A terceira e última particularidade diz respeito às possibilidades que a formalização de sistemas atômicos em Cálculo de Sequentes podem representar para a análise semântica de lógicas não clássicas em geral e de lógicas subestruturais em particular. Em síntese, a generalização nos permite distinguir entre sistemas atômicos fechados sob regras estruturais e sistemas não fechados, o que por sua vez permite a criação de estruturas que permitem a fácil manipulação de comportamentos "não-standard" observados nestas lógicas.

Dependo de quais restrições sejam impostas sobre os sistemas atômicos e quais cláusulas atômicas sejam utilizadas, as semânticas prova-teóricas talvez permitam a criação de semânticas simples e intuitivas para lógicas não clássicas. A título de exemplo, se permitirmos sistemas atômicos que não contenham todas as regras de corte atômico, é possível que em um sistema os sequentes $\emptyset \Rightarrow a$ e $a \Rightarrow \bot$ sejam deriváveis sem que o sequente $\emptyset \Rightarrow \bot$ seja derivável. Caso a derivabilidade de $\emptyset \Rightarrow a$ seja considerada condição necessária e suficiente pelas cláusulas semânticas para que a fórmula a seja válida/verdadeira e a derivabilidade de $a \Rightarrow \bot$ necessária e suficiente para que a seja inválida/falsa, a diferença entre modelos comuns e paraconsistentes poderia ser caracterizada pela presença ou ausência de regras de corte no sistema atômico. Também poderíamos considerar sistemas fechados ou não sob *weakening* atômico, contração atômica e até mesmo permutação atômica, o que permitiria a criação de semânticas simples para sistemas sintáticos obtidos a partir de restrições em cada uma destas regras.

Palavras-chave. Semânticas prova-teóricas; Lógica Intuicionista; Dedução Natural; Cálculo de Sequentes; Lógicas Não-classicas.

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On a way to visualize some Grothendieck Topologies

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Abstract

The *canonical Grothendieck topology* on $\mathbb{R}$, J_{can}, is easy to define, but the definition takes several steps: 1) for each open set $U \in \mathcal{O}(\mathbb{R})$ a *sieve on* U is a subset of $\mathcal{O}(U)$ that is downward-closed; 2) for each $U \in \mathcal{O}(\mathbb{R})$ we write $\Omega(U)$ for the set of all sieves on U; 3) we say that a sieve $\mathcal{S} \in \Omega(U)$ is *covering* when $\bigcup \mathcal{S} = U$; 4) for each $U \in \mathcal{O}(\mathbb{R})$ we define $J_{\text{can}}(U)$ as the set of covering sieves on U.

The "real" definition of Grothendieck Topology generalizes this definition of J_{can} in many ways: in particular, it starts with a category $\mathbf{C}$ instead of a topological space $(\mathbb{R}, \mathcal{O}(\mathbb{R}))$, and we can have many notions of "covering-ness" for the same category — they just have to obey the three axioms in [1], p.110.

In this presentation I will show how we can use some of the techniques in [2] to understand the general definition of Grothendieck Topology, and I will show how we can visualize all the Grothendieck Topologies on one of the Planar Heyting Algebras of [4]. Most of the diagrams in the presentation will be taken from [3].

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Two pure ecumenical natural deduction systems

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Abstract

Natural deduction systems, as proposed by Gentzen [1] and further studied by Prawitz [4], is one of the most well known proof-theoretical frameworks. Part of its success is based on the fact that natural deduction rules present a simple characterization of logical constants, especially in the case of intuitionistic logic. However, there has been a lot of criticism on extensions of the intuitionistic set of rules in order to deal with classical logic. Indeed, most of such extensions add, to the usual introduction and elimination rules, extra rules governing negation. As a consequence, several meta-logical properties, the most prominent one being *harmony*, are lost.

In [5], Dag Prawitz proposed a natural deduction *ecumenical system*, where classical logic and intuitionistic logic are codified in the same system. In this system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation and the constant for the absurd, but they would each have their own existential quantifier, disjunction and implication, with different meanings. Prawitz' main idea is that these different meanings are given by a semantical framework that can be accepted by both parties.

In this talk, we propose two different approaches adapting, to the natural deduction framework, [a] Girard's mechanism of *stoup* [2] and [b] Murzi's proposal [3] of combining Peter Schröder-Heister's higher-level rules [6] and Neil Tennant's idea of the $\perp$ as a punctuation sign [7]. This will allow the definition of a pure harmonic natural deduction system ($\mathcal{LE}_p$) for the propositional fragment of Prawitz' ecumenical logic. In the final part of the paper we show how to extend Murzi's approach to the first-order case.

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Some Many-Valued Logical Frameworks for Reasoning About Fiction

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Abstract

In view of the limitations of classical, free, and modal logics to deal with fictional names, we will present some logical frameworks that we see as promising ways of modeling contexts of reasoning in which those names occur. Specifically, we propose to evaluate statements in terms of factual and fictional truth values in such a way that, say, declaring 'Socrates is a man' to be true does not come down to the same thing as declaring 'Sherlock Holmes is a man' to be so. As a result, our frameworks are capable of representing reasoning about fictional characters that avoids evaluating statements according to the same semantic standards. The frameworks encompass several many-valued logics that differ according to (i) alternative ways one may interpret the relationships among the factual and fictional truth values; and (ii) whether or not statements in which fictional names occur are subject to some standard classical principles, viz., *explosion* and *excluded middle.*

Keywords. philosophy of fiction, fictional names, logic of fiction, many-valued logics.

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Proof systems for Geometric theories (PROGEO)

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Abstract

One of the advantages of using sequent systems as a frameworks for logical reasoning is that the resulting calculi are often simple, have good proof theoretical properties (like cut-elimination, consistency, etc) and can be easily implemented, e.g., using rewriting.

Hence it would be heaven if we could add axioms in mathematical theories to first order logics and reason about them using all the machinery already built for the sequent framework. Indeed, the general problem of extending standard proof-theoretical results obtained for pure logic to certain class of non-logical axioms has been focus of attention for quite some time now.

The main obstacle for this agenda is that adding non-logical axioms to systems while still maintaining the good proof theoretical properties it is not an easy task. In fact, as described in [7], if A, B are atoms and the axioms $\vdash A \supset B$ and $\vdash A$ are added to the sequent system LJ for intuitionistic logic [3], then the sequent $\vdash B$ can be derived using *cut*:

$$\dfrac{\dfrac{}{\vdash A} \qquad \dfrac{\dfrac{}{\vdash A \supset B} \qquad \dfrac{\dfrac{}{A \vdash A}\ \mathit{init} \qquad \dfrac{}{B \supset B}\ \mathit{init}}{A, A \supset B \vdash B}\ \supset L}{A \vdash B}\ \mathit{cut}}{\vdash B}\ \mathit{cut}$$

But it is easy to see that there is no proof of this sequent *without* cut. That is, the resulting system is not *cut-free*: applications of the rule *cut* can not be eliminated.

One way of circumventing this problem is by treating axioms as *theories*, added to the sequent context. This is already in Gentzen's consistency proof of elementary arithmetic in [2]. Now the derivations have only logical axioms as premisses, and cut elimination applies. In the example above, we can derive B from $A, A \supset B$ without a problem

$$\dfrac{\dfrac{}{A \vdash A}\ \mathit{init} \qquad \dfrac{}{A, B \vdash B}\ \mathit{init}}{A, A \supset B \vdash B}\ \supset L$$

But we can do better by transforming the axioms above into *inference rules*. In fact, if A, B are atomic formulas and C an arbitrary formula then, in the presence of $A \supset B$, if B proves C then A also proves C. On the other hand, in the presence of A, if A proves C, then C is provable (the A is irrelevant since it is *already there*). This induces the inference rules

$$\dfrac{\Gamma, B \vdash C}{\Gamma, A \vdash C}\ A \supset B \qquad \dfrac{\Gamma, A \vdash C}{\Gamma \vdash C}\ A$$

The sequent $\vdash B$ now has the (cut-free) proof

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\ \mathit{init}}{A \vdash B}\ A \supset B}{\vdash B}\ A$$

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In this talk, we will show a systematic way of adding transforming axioms into inference rules, and smoothly adding the resulting rules into well known sequent systems. The method is based on the notions of *focusing* and *polarities*, and it will be illustrated next for the case of *geometric axioms*.

First of all, given a (sequent calculus) proof system S, a *focused* version of it is a sound and complete proof system SF, where all proofs in SF correspond to normal form proofs in S. Such normalization relies in a two-phase construction of proofs: negative/positive, which consists solely of the application on invertible/non-invertible inference rules. The most well known focused systems for classical (LK) and intuitionistic (LJ) logical systems are LKF and LJF [4], respectively.

Focused systems come together with the polarity of formulas: atoms can be given negative or positive polarities, while the polarity of non-atomic formulas is given by the polarity of its outermost connective. For example, in LKF and LJF there are positive and negative versions of the conjunction/disjunction: $\wedge^+, \wedge^- / \vee^+, \vee^-$, the existential quantifier $\exists$ is always positive, while the implication $\supset$ and universal quantifier $\forall$ are always negative. In this way e.g., the formula

$$\forall x.(A \wedge^+ B) \supset C$$

is negative, while the subformula $A \wedge^+ B$ is positive. The is a very special class of polarized formulas, called *bipolars*. Intuitively, bipolars are formulas in which the number of flipping (nested) connectives polarities is at most one. The formula above is bipolar since, starting from the positive $A \wedge^+ B$, we flip only once for building the negative $(A \wedge^+ B) \supset C$.

Geometric axioms are first-order formulas that can be converted into (natural deduction/ sequent) inference rules having "a certain simple form in which only atomic formulas play a critical part", as described by Simpson [8]. And this "simple rules for atomic formulas" motto seems to be the core of success in this endurance in the approaches/extensions present in the literature [1]. Our claim is that the combination of bipolars and focusing is the real essence of "simple rules for atomic formulas".

Definition 1. A *geometric implication* is a first-order formula having the form

$$\forall \overline{z}(P_1 \wedge \ldots \wedge P_m \supset \exists \overline{x}_1 M_1 \vee \ldots \vee \exists \overline{x}_n M_n),$$

where each P_i is an atomic formula, each M_j is a conjunction of atomic formulas $Q_{j_1}, \ldots, Q_{j_{k_j}}$, and none of the variables in the lists $\overline{x}_1, \ldots, \overline{x}_n$ are free in P_i. A *geometric theory* is a finite set of geometric implications. We shall also assume that if the list of variables $\bar{x}_i$ is empty then M_i is just an atom: otherwise, this formula can be written as a conjunction of geometric implications.

Observe that geometric formulas are *negative*, and can be polarized as

$$\forall \overline{z}(P_1^{\pm} \wedge^{\pm} \ldots \wedge^{\pm} P_m^{\pm} \supset \exists \overline{x}_1 M_1^{\pm} \vee^{\pm} \ldots \vee^{\pm} \exists \overline{x}_n M_n^{\pm})$$

Not all such polarizations give rise to bipolars, though.

A simple example of a geometric implication is the *transitivity* axiom, stating that, for a binary relation $R \subseteq W \times W$ on a non-empty set W, for all $x, y, z \in W$, if x is related to y and y is related to z then x is related to z

$$4 = \forall x, y, z.(R(x, y) \wedge R(y, z)) \supset R(x, z)$$

For polarizing this formula in LKF or LJF, we can give to the atomic predicate R and the conjunction *positive* or *negative* polarities. We then obtain the following four poralized formulas (all of them bipolars)

$$4^{\pm} = \forall x, y, z.(R(x, y)^{\pm} \wedge^{\pm} R(y, z)^{\pm}) \supset R(x, z)^{\pm}$$

In this talk, we will show the systematic method presented in [5] for transforming axioms like $4^{\pm}$ into corresponding focused inference rules (called *bipoles*) in LKF/LJF. We will then show how to transform each of these rules into the respective unfocused version in LK/LJ.

For this example, focusing on each and all of the possible formulas $4^{\pm}$ in LJF (it holds also for LKF) will produce the following two inference rules (bipoles)

$$\frac{R(x,z), \Gamma \vdash C}{R(x,y), R(y,z), \Gamma \vdash C}\ 4_{GRS} \qquad \frac{\Gamma \vdash R(x,y) \quad \Gamma \vdash R(y,z)}{\Gamma \vdash R(x,z)}\ 4_{RR}$$

The rule 4_{GRS} appears in [6] and corresponds to backward-chaining, while the rule 4_{RR} is the transitivity rule studied in [10], corresponding to forward-chaining. This implies that these works are different faces of the same coin, the latter being minted from focusing and polarization. Finally, observe that we address these issues with a uniform presentation in both classical and intuitionistic first-order logics.

We will apply this method for specifying Tarski's geometry [9], a complete first-order axiomatization of Euclidean plane geometry.

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A Teoria da Computabilidade Clássica desde um Ponto de Vista Intuicionista

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Resumo

Nossa apresentação versará sobre a axiomatização intuicionista da Teoria da Computabilidade Clássica proposta por Fred Richman [1], (cf. também Bridges e Richman [2]). Essa versão intuicionista, por sua vez, baseava-se em uma formulação axiomática anterior (clássica) oferecida por Eric Wagner [3]. Após fornecer uma caracterização intuicionista para a noção clássica de "função parcial", a axiomatização de Richman extrai a teoria inteira da Computabilidade Clássica a partir de um único axioma que assere que aquelas funções parciais seriam enumeráveis. A axiomatização de Fred Rincham nos oferece um acesso privilegiado a um tema muito controverso e importante no contexto das relações entre a lógica e a matemática intuicionista, por um lado e a clássica, por outro: a maneira como o intuicionista encara vários resultados clássicos ligados à noção de "Computabilidade" e mesma à Tese de Church.

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Quadratic Extensions of Special Hyperfields and the Aranson-Pfister Hauptsatz

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Abstract

The uses of K-theoretic and Boolean methods [4–6] in first-order theories of quadratic forms has been proved a very successful approach, see for instance, these two papers of M. Dickmann and F. Miraglia: [2] where they give an affirmative answer to Marshall's Conjecture, and [3], where they give an affirmative answer to Lam's Conjecture.

These two central papers makes us take a deeper look at the theory of Special Groups by itself. This is not mere exercise in abstraction: from Marshall's and Lam's Conjecture many questions arise in the abstract and concrete context of quadratic forms and are still open in the context of (non reduced) special groups. Here we use multialgebras to attack one of them: the Arason-Pfister Hauptsatz.

The concept of multialgebraic structure has been studied since the 1930's; in particular, the concept of hyperrings was introduced by Krasner in the 1950's. In more details: a multivalued n-operation on a set A is just a function $A^n \to P(A) \setminus \{\emptyset\}$, thus every n-ary operation on A can be seen as an n-ary multioperation on A and n-ary multioperations on A can be identified with $n+1$-ary relations $R(x_1, \cdots, x_n, x_{n+1})$ satisfying some $\forall\exists$-axiom.

Some general algebraic study has been made on multialgebras, in particular first steps towards a "Multi-Universal Algebra" were given in [11]; a collection of results focused on multi-algebraic semantics for non-classical logics in provided by the survey [8]. More recently the notion of multiring have obtained more attention: a multiring is a lax hyperring, satisfying an weak distributive law, but hyperfields and multifields coincide. Multirings have been studied for applications in abstract quadratic forms theory [7, 10, 14, 15] and tropical geometry [9]. A more detailed account of variants of concept of polynomials over hyperrings is even more recent [1, 9].

In the present talk we attack the introduce the Arason-Pfister Hauptsatz for non reduced special groups via the concept of quadratic extension of hyperfields. This is based on a ongoing project [12, 13].

Keywords. multialgebras, model theory, special groups, Hauptsatz.

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Propagation of classicality in logics of evidence and truth

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Abstract

The logics of evidence and truth (*LET*s) are part of an evolutionary line that starts in the 1960s with the seminal work of da Costa, which was based on two main ideas: (i) to divide the sentences of the language into two groups, one subject to classical logic and the other subject to a non-explosive logic, and (ii) to express the metatheoretical notion of consistency at the object language level. These ideas were further developed in the 1970s and 1980s, among others, by Loparic, D'Ottaviano and Alves. More recently, from 2000 onwards, the *LFI*s (logics of formal inconsistency) are part of the same evolutionary line, investigated, among others, by Carnielli, Coniglio and Marcos. *LET*s, however, have a specific motivation: to express the deductive behavior of positive and negative, conclusive and non-conclusive evidence. The underlying idea is that when dealing with conclusive evidence we use classical logic, and with non-conclusive evidence we use a paraconsistent and paracomplete logic based on the Belnap-Dunn four-valued logic. According to the intended interpretation, a pair of contradictory formulas A and $\neg A$ express the simultaneous presence of positive and negative evidence for A, $\circ$ is a classicality operator, and $\circ A$ means that there is conclusive evidence for either A or $\neg A$. Thus, classical logic is recovered for propositions for which the available evidence is considered conclusive. Propagation of classicality is how the classical behavior is transmitted from less complex to more complex sentences, and vice-versa. In this paper, we present a natural deduction system for the logic $LETF+$, an extension of the logic $LETF$ [1], with introduction and elimination rules conceived to express propagation of classicality, along with a sound, complete, and decidable six-valued semantics. (Joint work with Marcelo Coniglio)

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Semiótica, não-monotonicidade e a diferença entre lógica e matemática segundo C. S. Peirce

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Resumo

É famosa a identificação, por Peirce, da semiótica com a lógica, com as seguintes palavras: "Em seu sentido geral, a lógica é, como acredito ter demonstrado, apenas um outro nome para semiótica, a doutrina formal, ou como que necessária, dos signos." [1, CP 2.227]. Tal definição, apesar de bem explicada pelo seu autor e de ser incontavelmente citada, nem sempre é bem compreendida. Neste trabalho, pretendo indicar alguns pontos básicos para seu entendimento. Em primeiro lugar, é preciso recuperar o lugar reservado por Peirce à lógica na sua classificação das ciências. Peirce distingue uma matemática da lógica, ou lógica exata (conforme Schroeder), como um ramo da matemática [4]. Assim, a matemática é a primeira, a sumamente abstrata e a mais geral das ciências da descoberta, definida pela atividade do matemático: um matemático é quem constrói mentalmente estados de coisas hipotéticos e extrai dedutivamente consequências desses estados de coisas, por meio de operações com diagramas. Daí que, como parte da matemática, a lógica exata não se sobreponha exatamente à semiótica. Esta, por sua vez, é uma parte da filosofia, um sub-ramo da investigação filosófica. A filosofia seria a segunda ciência da descoberta, menos geral apenas do que a matemática e diferente desta porque os filósofos tomam como objeto de estudo a experiência humana, em geral, algo de que os matemáticos podem prescindir, em certo sentido [2]. Por isso, a filosofia é uma investigação com três diferentes ramos, a saber: fenomenologia, ou *faneroscopia*, ciências normativas - estética, ética e lógica, ou *semiótica* - e metafísica. Como a terceira das ciências normativas, a semiótica de Peirce tem como tarefa investigar a natureza, a gênese e os usos dos signos em geral. Como tal, a semiótica não se limita ao estudo de formas necessárias de raciocinar, mas estende-se ao campo do que hoje em dia pode ser denominado de lógica da não-monotonicidade [3]. É esse aspecto da semiótica que permite distinguir mais claramente a diferença entre a lógica e a matemática e seu estatuto como ciências heurísticas, aquelas, segundo Peirce, que nos permitem conhecer o que desconhecemos.

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Quantale modules and deductive systems: where we are and where we are heading

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Abstract

An approach to abstract deductive systems by means of quantale modules was proposed by N. Galatos and C. Tsinakis in [3] and eventually developed by the present author [5, 6, 8]. However, apart from occasional references to logic appeared, for example, in [1, 2, 4], many concrete applications of the order-theoretic apparatus are still lacking.

More precisely, in [3] the authors showed that propositional deductive systems can be fully described by means of their lattices of theories which are modules over the powerset quantales of the monoids of substitutions of their languages. Among other things, they proved that, given two deductive systems over the same language, the existence of a quantale module homomorphism between the corresponding modules of theories is a sufficient condition for the existence of an interpretation of one system into the other. The present author extended that result in two directions [6], namely, to the case of systems with different underlying languages and to both weaker and stronger forms of interpretations.

Further possibile applications were pointed out by all of us in those papers, but most of them have not been concretely explored in details. In the present talk, we shall precisely address this issue.

We will show various techniques for combining different languages and deductive systems thus obtaining a new one which encompasses the consequence relations of all of the initial systems. In particular, we will prove that the whole machinery is flexible enough not only to handle different situations but also to propose alternative constructions for each single case. We can briefly resume such constructions as follows

- The failure of the amalgamation property for quantales is an obvious consequence of the one for semigroups and monoids. However, we prove that quantales of substitutions of propositional languages can actually be amalgamated, and in a very easy way.
- Two systems with the same underlying language can always be merged either using $\mathcal{Q}$-module (possibly amalgamated) coproduct or by "doubling" the language. Module coproduct gives a system on some kind of two-sorted syntactic constructs and has the advantage of working in a pretty standard way, in the sense that its concrete implementation does not depend on the particular systems at hand. On the other hand, the construction based on the coproduct of quantales is definitely more effective when dealing with systems of the same type (both on formulas, equations, or sequents).
- Merging systems over different languages is obviously more laborious using module (possibly amalgamated) coproduct. On the contrary, the construction with quantale coproducts is essentially the same as the single-language case, but the capacity of taking into account possible isomorphic pieces of the modules of theories depends on the existence or not of some (possibly partial) translation and interpretation.

The results above are contained in [9].

Last, we shall discuss possible directions for future works, some of which are being already investigated by the author.

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An ontology-based microservice approach for data interoperability

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Abstract

Data interoperability [1] is the processing of data from multiple sources or with distinct formats with the focus of delivering to the end-user unified and simple data. It can also represent a way of reading multiple bases, in a way that the user does not need to know the topology of the data.

This work discusses an architecture to make alignment of entities relying on micro-services. Alignment is the process of identifying semantically equivalent terms in two or more bases. It leads to the possibility of more complex and robust queries and inferences. The modules present in the work are grouped in four steps: (i) preparation, (ii) parameter matching, (iii) entity matching, and (iv) post-analysis. Each group composes a general functionality. This architecture was developed in the context of the project Interopera–PDPA and is being used by the city of Niterói–RJ, Brazil.

The preparation group is intended to provide means of reading and adjusting the provided bases to improve the alignment. The modules that are present in this group receive from the user two or more CSV files, a group of service options, and a set of information from these. Then the base is created from the parameters and entities found in the files and on the sequence adjustments are made based in the information provided. In the end, the base and the options are sent to the next group.

The parameter matching group is intended to provide an alignment of the parameters found in the bases. The modules that are present in this group receive the bases that will be aligned and the options of which modules will be selected to do the alignment, the selected module then compares every parameter between the bases, and if the comparisons pass the module threshold they're a match. When a module finishes its alignment the group searches for another module in the options to complement the alignment, this approach provides an accessibly means of expanding with new modules since each module doesn't interfere with each other. The modules present in this group are the translation [7], text distance and synonym modules.

The entity matching group is intended to provide an alignment of the entities found in the bases. The modules that are present in this group receive the bases with the aligned parameters from the previous group and the options of which modules will be selected to do the alignment, the selected module then compares every entity of the parameters that were matched between the bases but for an entity to be considered a match between them it needs to be a match for every matched parameter in the bases. This group provides a way to choose the best modules to make the alignment of their entities with the addition of two new modules. The new modules present in this group is the exact and deep matcher [6] module.

The post-analysis group is intended to provide a means of generating an output with the aligned bases and a way to make inferences in these bases. The modules that are present in this group receive the aligned bases from the previous group and make a table file with the result of the alignment, then the group branches in three paths, which are described as follows.

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The first path provides a means to further enhance the aligned result by running another process of aligning the entities with the machine learning algorithm. The second path provides a means of making inferences in the aligned bases by allowing the user to make queries using Boolean operators and in the SQL language. The third step takes the aligned base and makes another output with a map of the words used to the Suggested Upper Merged Ontology (SUMO) and use this map to link the bases with the SUMO ontology through the Sigmakee tool after that the user can then make inferences about their bases. This abstract will be focused in the third path and its features.

Ontology [4] can have multiple definitions, but in the context of this project, an ontology is a formal and well-defined model of knowledge. There are different types of ontology, like upper or hybrid. An upper ontology uses general concepts for any domain of knowledge, and with this property, SUMO tries to represent the real world. It also features an automated way of reasoning about their data, this feature makes it possible to do consistency checks and improves satisfiability requirements.

The Suggested Upper Merged Ontology [3] (SUMO) is currently the largest free ontology available. All entries are mapped with the WordNet relation that can be subsumed, equivalent, or an instance. For example, "code" has an equivalent map with "ComputerProgram".

The Wordnet [2] constitutes a large lexical database for a language. This database lists words such as synonyms, antonyms, declensions, among others.

To make the map the tool searches for all parameters found in the aligned bases and with it makes a relation of the parameter with a term in the SUMO ontology. For every term mapped in this way an instance of the entities related with the parameter that was mapped is added to the term in the ontology, after the instances were added a relation is created between them to simulate the relation of the entity.

With the bases mapped to the ontology, the user can take all the power of the reasoner that the SUMO ontology has, thus granting the user the possibility to make inferences that were previously unavailable in the bases. Some basic inferences that can be done in the ontology are listed as follows.

- Retrieve an entity - instance $\times$ entity
- Retrieve details of a term - instance $\times$ term
- Retrieve every entity with a property $\times$ hasProperty some property

To work with these inferences the users have two choices making the inferences with the tools that the architecture will provide or using the interface that the Sigmakee tool provides. The Sigmakee [8] is a tool to work with the SUMO ontology and with the interface the user can use all the features of the tool. This interface can also be seen as a graph generated by the Graphviz tool [5]. An example of how the ontology can help the user to improve how they can explore their data is presented as follows.

Consider a city that has its data broken into two systems. The first one contains information about the families that live in the city and the second one contains the details of every person in the city. These systems were designed by distinct teams and rely on different technologies, therefore they do not interact with each other. The city is now trying to get complete information about the families with every detail that they can get about them, so they could improve their campaigns in the area.

Using this architecture the city can unite every person to a family automatically and after the data has been aligned and put in the ontology the city could make inferences about the persons that have more than one family, areas with more juvenile persons, get persons that are sisters and brothers between them and much more. With the ontology, the city could also make inferences that were not available with only the database data like making inferences about the finances with the help of the finance ontology present in the SUMO or reading characteristics of the individuals of a family with the person ontology that is present as well.

This structure grants the users new ways of reading and exploring their data with the help of the content already present in the ontology granting them new data that they could use to improve their bases or to make new inferences about their content.

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Problemas e seus tipos na Geometria Euclidiana

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Resumo

O *Livro I* dos *Elementos de Euclides* [1] começa com três *propositiones* que pedem a solução de três problemas. Elas não constituem uma asserção de propriedades ou relações entre objetos geométricos, mas clamam por ações que devem ser executadas para a obtenção de determinados objetos. A solução desse tipo de problema é normalmente designada como *construção* e os alicerces de tal construção são as definições, as noções comuns e os postulados da geometria euclidiana. Além disso, cada construção deve vir acompanhada da resolução de um segundo problema. Este consiste em demonstrar que a solução oferecida é *correta*, ou seja, que a construção apresentada produz de fato aquilo que havia sido demandado na formulação do problema.[1]

No espírito do século XIX, a formulação da geometria euclidiana passou por uma transformação na qual todas as suas *propositiones* foram reapresentadas como teoremas, ou seja, como asserções de propriedades ou relações entre os objetos da geometria. Essa mudança exigia que alguns dos princípios fundamentais da teoria fossem reformulados como asserções, notavelmente os três primeiros postulados. Desde esse ponto de vista, a distinção entre postulados e noções comuns tendia ao apagamento sendo tratados identicamente como *axiomas*. Esse apagamento não é propriamente novo, uma vez que Aristóteles pode ser lido, e muitas vezes o foi, como um autor que não haveria dado importância às distinções entre noções comuns e postulados.

A nosso ver, a geometria euclidiana pode ser lida de um ponto de vista inverso desse último: as *propositiones* podem bem ser interpretadas como problemas, o que chamaremos de interpretação *problemacional*. Essa reviravolta terá alguma utilidade se ela permitir distinguir nuances de outro modo apagadas na interpretação assercional das *propositiones*.

Defendemos a ideia de que a formulação original das *propositiones* poderá ser mantida sem que se percam as razões do seu aporte e do seu ordenamento dentro dos livros dos *Elementos*. Por si mesmo, isto já seria uma vantagem, uma vez que a interpretação assercional produz distorções na tradução das *propositiones* em teoremas. Ou, pelo menos, a formulação assercional parece alterar de modo substancial o propósito original da formulação.

A título de exemplo, consideremos o seguinte caso:

Propositio III.1 – Achar o centro do círculo dado.

Claramente, ela pede que se efetue uma construção. Ora, transformar esta proposição em uma asserção não pode consistir em afirmar que "Dado um círculo ABC, existe um ponto G que é o centro deste círculo". Isso se dá por uma simples e excelente razão. As definições de Euclides já caracterizam o círculo como uma figura geométrica que possui um único centro! A *propositio III.1* só fará sentido se admitirmos que é concebível estar na presença de um círculo sem no entanto saber exatamente onde seu centro está localizado. O procedimento de construção demandado permitiria descobrir essa informação. Caso queiramos reinterpretar essa *propositio* como uma asserção de existência, ela só poderá sê-lo feita se a transformarmos na asserção da existência de um procedimento para encontrar o centro! Porém, em vista da consistência e da harmonia, esse padrão deveria ser empregado para interpretar todas as *propositiones* que requeiram construção.[2]

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[1]A exigência da solução deste problema subsequente é uma característica conspícua do labor matemático.

[2]Sob esse ponto de vista, haveria boas razões para afirmar que a geometria euclidiana é, em essência, uma teoria construtivista, dado que as provas de suas *propositiones* regularmente exibem um procedimento de construção.

A interpretação existencial simples pode ser e tem sido questionada, por exemplo por Harari [2].

A leitura problemacional assume que a formulação dos postulados e noções comuns seja feita de modo preferencial como problemas. E, uma vez que as *propositio*-problemas devem ser resolvidas com base nos princípios da geometria, pelo menos alguns destes últimos devem também ser interpretados como problemas. Problemas de uma natureza peculiar. Problemas que assumiremos estarem resolvidos, ou seja, problemas para os quais possuímos uma ou mais soluções todas elas produzindo resultado equivalente.

Os três primeiros postulados do Livro I dos Elementos podem naturalmente ser lidos como problemas. A seguinte explicitação indica como isso pode ser feito:

I) [Como?] traçar uma reta de um ponto qualquer até um ponto qualquer;

II) [Como?] prolongar uma reta dada até um ponto qualquer;

III) Dado um ponto tomado como centro e uma reta, [como?] traçar um círculo tomando essa reta como raio [e aquele ponto como centro].

O fato de tomá-los como postulados equivale, como já apontamos, a assumir que esses problemas estão resolvidos. Todavia, os postulados 4 e 5 demandam exame mais delicado.

As noções comuns são exemplos claros de regras inferenciais na geometria euclidiana. Elas são genéricas, pois não está especificado de qual tipo de igualdade ou desigualdade se trata. A igualdade pode ser predicada de diferentes objetos que pertencem a categorias distintas. Em outros termos, é uma noção comum a categorias distintas.

As três primeiras *propositiones* do *Livro I* demandam a construção de um objeto geométrico. A primeira, um triângulo equilátero a partir de uma reta dada; a segunda pede que se transfira sobre um dado ponto como extremidade uma dada reta; a terceira pede que se determine a subtração entre duas retas. Cada uma destas construções vem apresentada por um discurso e complementada por uma prova de que o objeto obtido tem as propriedades desejadas. Podemos assim dizer que, de modo geral, os problema de construção na geometria comportam a solução de dois problemas em sequência: o problema da construção propriamente dito e, em seguida, o problema de demonstrar que a construção efetivamente tem as propriedades requeridas.

A *Propositio I.4* não pede como solução uma construção. Essa *propositio*, a primeira do seu gênero, vem formulada como uma asserção condicional, um teorema. Mas ela pode perfeitamente ser lida como um problema. Como problema ela seria reformulada do seguinte modo:

(i) *Questão*: quais as condições mínimas requeridas de dois triângulos para que possam ser ditos idênticos?

(ii) *Enunciação da condição*: que um triângulo tenha dois lados iguais a dois lados do outro e que o ângulos entre esses lados seja o mesmo em ambos triângulos. Adicionalmente, essa condição seria também suficiente para determinar que os demais ângulos dos dois triângulos se equivalem, assim como a base deles.

Desde essa perspectiva, o problema de estabelecer a igualdade ou congruência de dois triângulos dados, reduz-se ao problema de estabelecer a igualdade de um par de lados e do ângulo compreendido entre eles em um e outro triângulo. A relação de redução equivale a uma relação lógica entre enunciados assercionais. Ou seja, para demonstrar a redutibilidade entre os dois problemas devemos supor a condição envolvendo os dois lados e o ângulo que eles compreendem e deduzir a igualdade ou congruência dos triângulos. Desde essa perspectiva, uma ideia similar pode ser aplicada ao quinto postulado como um problema. Interpretar o quarto postulado como um problema é um assunto delicado e será tratado em outro momento.

O *Quinto Postulado* é formulado do seguinte modo:

> E, caso uma reta, caindo sobre duas retas, faça os ângulos interiores e do mesmo lado menores do que dois retos, sendo prolongadas as duas retas, ilimitadamente, encontrarem-se no lado no qual estão os menores do que dois retos.

Este postulado pode ser reformulado como o problema de saber quando, por meio de extensão (cf. o *Postulado I.2*), as duas linhas se encontrarão. A condição para que isso aconteça é a de que uma terceira reta cortando essas duas forme ângulos de um mesmo lado que sejam menos do que dois retos. As linhas se encontrarão se adequadamente estendidas exatamente deste lado e não do outro. Em outros termos o postulado estabelece uma condição suficiente para que duas linhas retas dadas não sejam paralelas. Ou ainda, estabelece uma condição necessária para que duas retas sejam paralelas: quando os dois ângulos internos com a terceira sejam iguais a dois retos.

Este postulado será utilizado pela primeira vez na prova da *Propositio I.29*, mas o tema do paralelismo aparece antes, já na *Propositio I.27*. Na verdade, tanto a *I.27* quanto a *I.28* apresentam duas condições suficientes para o paralelismo de duas retas. Um caso envolve a igualdade dos ângulos alternos com a terceira reta, no outro a igualdade do ângulo exterior com o interior envolvendo a terceira terceira reta. Ambas *propositiones* reduzem o problema de determinar o paralelismo de duas retas ao de saber se uma dada igualdade com a terceira se dá. Curiosamente, a prova destas reduções não envolve o quinto postulado, apenas a definição de paralelismo. O postulado poderia ser reformulado do seguinte modo: se os alternos não são iguais, então as retas se encontrarão quando devidamente estendidas.

Uma questão que naturalmente surge após as considerações acima é: existiriam outros problemas demandando a busca de uma condição para que ocorra um determinado estado de coisas? Se a resposta for positiva, fica claro que a interpretação dada à *Propositio I.4* não constitui solução *ad hoc*, e indica que uma investigação dos diferentes tipos de problemas contidos nos *Elementos de Euclides* é tarefa relevante.

Saltando para o *Livro IX*, as *propositiones IX.18* e *IX.19* parecem heterogêneas com respeito à formulação das demais:

> *Propositio IX.18* - Dados dois números, examinar se é possível achar a mais um terceiro em proporção com eles.
>
> *Propositio IX.19* - Dados três números, examinar quando é possível achar a mais um quarto em proporção com eles.

Em ambos casos pede-se uma investigação. No primeiro, investigar se é possível obter um número com determinada propriedade a partir de dois outros números dados. Cada número é representado por um segmento de reta. No segundo, a investigação é proposta de modo ligeiramente diferente. Pede-se investigar quando é possível encontrar um número proporcional a três números dados.

Dado um problema, não sabemos de antemão se o problema pode ser solucionado ou não. Por sua vez, em sentido amplo solucioná-lo envolve um de dois casos. Uma solução positiva, quando garante-se a existência da solução, e ela é exibida. Ou uma solução negativa, quando se mostra que uma solução positiva é impossível. Há ainda, está claro, uma terceira possibilidade, quando o problema é considerado não-resolvido, ou seja, quando não se obteve resposta positiva nem resposta negativa.

Nas duas questões acima pede-se uma investigação de possibilidade.

Está claro que antes de ler as provas dessas proposições, já esperamos que a resposta seja positiva em ambos casos, de outro modo a *propositio* não estaria listada no livro *IX*. Em *IX.18*, mostra-se que existe uma resposta positiva à questão quando os números dados, A e B, não são primos entre si e A mede o quadrado de B. Se a condição não se cumpre, não haverá um terceiro proporcional. Mas a resposta não é formulada como uma asserção condicional como normalmente esperaríamos, aliás nem é formulada explicitamente. Já em *IX.19* pede-se investigar quando estaria garantida a existência. O raciocínio lógico aponta que quando os números dados, A, B e C, são continuamente proporcionais e, além disso, A não é primo com respeito a C, então, se A mede D – onde D é obtido multiplicando B e C – A, B, C e D são proporcionais. E mostra-se que a conjunção dessas condições é suficiente e necessária para a proporcionalidade. Novamente, a *propositio* não tem uma formulação teoremática, mas que é perfeitamente bem compreendida como problemática. A solução em ambas consiste em apontar sob que condição haverá um terceiro/quarto proporcional.

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Quantum Algorithms for Multiplicative Linear Logic

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Abstract

Quantum computing is the area that studies the development of algorithms and software that utilize the movement of sub-atomic particles to represent and process information. Quantum computing is capable of creating algorithms with better asymptotic complexity than any classical counterpart, and it has applications in many fields, such as cryptography, chemistry and machine learning. In the last decades, quantum computing has seen considerable improvement, with the development of quantum computers with over 100 qubits, and it has been seen by many as the next big step in the world of informatics.

Superposition is the ability of a quantum system to be in multiple states at the same time until it is observed. Several experiments make use of this property, the most recognizable being Schrodinger's cat and the double slit experiment. A quantum state is described by the combination of all possible states in the superposition. Quantum bits or, more commonly, *qubits*, are the quantum computing equivalent to classical computing bits, and a quantum state is composed of one or more qubits. The main difference from bits to qubits is that, while a bit has a value of 0 or 1, a qubit can be in a superposition of both states until it is observed, which causes the actual value of the qubit to collapse to either 0 or 1. This means that while n bits can store only one value at a certain point in time, n qubits can store 2^n values in the superposition, an exponential increase. Taking advantage of the superposition, it is possible to perform parallel operations on every state. This is called *quantum parallelism*, and is one of the sources for the speed-up of quantum algorithms.

Measurement is the act of observing a quantum state, causing it to collapse to a classical value. Since this is an irreversible operation, it is usually the last step of a quantum algorithm - measuring a quantum state destroys the superposition. This is an inherently probabilistic process - when a qubit is measured, it has α probability of returning 0 and β probability of returning 1. α and β are called *amplitudes*, and $\alpha^2 + \beta^2 = 1$ is always true. The most adopted way of representing quantum states is Dirac's Bras and Kets notation, which will represent a qubit as $\alpha |0\rangle + \beta |1\rangle$.

One of the most famous quantum algorithms is Grover's search algorithm [1]. Grover's search algorithm (GSA), was developed by Lov K. Grover in 1996 and its goal is to search for an element in an unordered database. Assuming a database with $N = 2^n$ items $i^1, i^2, ...i^N$, where each item can be represented by a binary string of length n, the objective is to find the searched item i that fulfills some condition C. The GSA has a time complexity of $O(\sqrt{N})$. This outperforms the classical algorithm, which has the complexity of $O(N)$ since we potentially need to check every member of the database. Even if we take into account a probabilistic analysis, we remain with a complexity of $O(N/2)$ - on average, we need to verify half of the database before finding the correct value. The GSA has been proven to be asymptotically optimal [3] and it is used in a wide array of applications, such as data processing, machine learning and others. The GSA can be divided into three parts: initialization, oracle and amplification. In the initialization, the database is put on the desired state, which is usually the equal superposition state:

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$$|\Psi\rangle = |0^n\rangle \otimes H^n = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$$

After the initialization, the oracle is applied. The oracle is an externally provided black box function that will mark the searched value in the database. After the oracle, the diffuser is applied, increasing the probability of the desired state being measured. These three phases together are called the Grover iteration, and this has to be repeated $\sqrt{N}$, hence the described complexity. There are several quantum computing models, such as quantum annealing, topological quantum computing, one-way quantum computing and the one we will be using throughout this project - gate-based quantum computing. Gate-based quantum computing uses quantum gates to build quantum circuits. Quantum gates are the basic building blocks to construct quantum circuits. They are operators that affect at most 2 qubits, and they must be reversible, that is, based on the output on must be able to retrieve the input and for any gate A, $AA^{\intercal} = \mathbb{I}$. This creates a series of limitations, since several classical logic operators, such as the $\wedge$ and the $\vee$, are not reversible. In quantum circuits, each horizontal line represents a qubit, and the horizontal axis is time. Gates are represented by blocks in the circuit attached to the affected qubits.

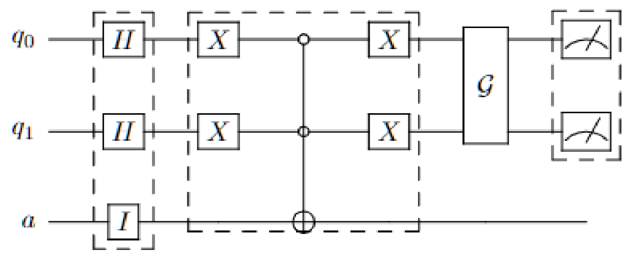

Figure 1: Circuit implementation of a single Grover's search algorithm iteration

An example of a quantum circuit representing a single iteration of the GSA can be seen in figure 1. The circuit has three qubits; q_0 and q_1 represent the search space of $N = 2^n = 4$ values, while a is an *ancilla* qubit, that is, a qubit used for specific computations that is later discarded without being measured. The first group outlined in the figure is the initialization part. For that, a Hadamard gate is applied to the database qubits to leave them in equal superposition. The Hadamard is a single qubit gate and is one of the fundamental gates in quantum computing. It takes a qubit from the state $|0\rangle$ or $|1\rangle$ to the balanced states, $\frac{1}{2}|0\rangle \pm \frac{1}{2}|1\rangle$, creating superposition. The initial step of most quantum algorithms consists in applying the Hadamard gate to one or more qubits in the state $|0\rangle$ to create a balanced quantum state and then perform the operations. The ancilla qubit is put in a specific state for it to be used as a target of a controlled operation in order to cause a phase kickback, which will be used to mark the desired state. The second group represents the oracle part where the correct state is marked. In this oracle, the qubits must be in the state $|11\rangle$ when they reach the multi-controlled operation, represented by the vertical line in the circuit. In our figure, the correct state is the $|00\rangle$. This is the case because the X gate is the amplitude flip, taking a qubit from $\alpha|0\rangle + \beta|1\rangle$ to the state $\beta|0\rangle + \alpha|1\rangle$. This will mean that the only state that will pass the multi-controlled operation while being $|11\rangle$ is the $|00\rangle$, since both qubits will be flipped. The X gates after the multi-controlled operation are used to bring the marked correct state to the original state of $|11\rangle$. The G block represents the Grover diffuser, where the marked state's amplitude is increased in order to increase the probability of it being measured, and the last group is composed of the two measure gates, that destroy the quantum state by observing it and return a classical value.

Linear logic was invented in 1987 by J. Y. Girard [2]. Its initial motivation was to unify in a single system both Classical and Intuitionistic systems with their semantic traditions: Tarski semantics, which focuses only on the denotation of a logical sentence and its sytnax, and Heyting semantics, which treats the logical sentences as abstract constructs that represent a proof. One of Linear Logic's new features is the division into two different logical connectors of conjuction and disjuction. The classical $\wedge$, for example, has its additive form & (with) and its multiplicative form $\otimes$ (tensor). This division allows for the decomposition of the intuitionistic implication, which in turn allows both the classical and non-classical approaches to coexist in the same system. Linear Logic also introduced the concept of formulas as resources, not allowing the traditional rules of weakening and contraction, unless if accompanied by specific modals. In Linear Logic, if we have a proof $A \to B$ and a proof of A, both proofs are consumed to generate B. This sensitivity to resource use has made Linear Logic quite popular for its applications in computer science. It also has a sequent calculus proof system. In this context, the algorithm for finding a cut-free proof in the multiplicative only version of Linear Logic has a worst-case time complexity of 2^k, where k is the number of atomic formulas. The subset of intuitionistic linear logic that deals only with the multiplicative connectives is called (intuitionistic) multiplicative linear logic (IMLL).

The main objective of this work is to develop a quantum proof search algorithm for sequent calculus in variations of Linear Logic. As result, we have three different solutions with small variances which we will analyse. All solutions make use of Grover's search algorithm. This solution follows Alsing's entangled database search [4] using the GSA. The main feature of Alsing's algorithm is that, instead of using $A = H^{\otimes n}$ to prepare the equal superposition state, it chooses A in order to encode an arbitrary list of pairs $\{s, t\}$. Thus, the algorithm's input is an entangled database with two sides, each side having one part of the pair. Every entry on the left side is entangled with an entry on the right side. In the GSA, it is necessary to know the searched value to construct the oracle. On Alsing's, on the other hand, one can construct the oracle based on a known entry s_1 on the left side, apply the GSA, and then measure the right side, recovering the unknown value t_1 entangled with s_1. The second solution is also for $\otimes$-only Linear Logic but uses a more standard GSA application with the Oracle as a black box, encoding the whole possible tree of sequent splits in the quantum database to be searched. The third solution is an extension of the second one, incorporating also the linear implication ($\multimap$). In this paper, we will describe and analyze the three algorithms in detail, compare them and present the results. The algorithms were implemented in IBM's Qiskit and tested in both quantum simulators and actual quantum devices.

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O Jogo de Cartas Lógicas de Shiver

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Resumo

Anthony Shiver (2013) desenvolveu dois jogos de cartas lógicas para a prática da derivabilidade no contexto da Lógica Sentencial Clássica (LSC). Embora ele tenha apresentado os contornos gerais desses jogos, muitos detalhes do desenho deles estão faltando. Neste trabalho proponho uma metodologia para o desenho detalhado do primeiro dos jogos de Shiver. Esse jogo utiliza um baralho de cartas lógicas, ou seja, em cada carta do baralho está inscrita uma sentença da LSC; o carteador distribui inicialmente três cartas comunitárias – as cartas de suposição – e cartas para cada jogador; um jogador pode descarregar uma de suas cartas se, e somente se, ela é derivável das cartas comunitárias; vence a partida quem primeiro descarregar todas as suas cartas ou, caso isso não ocorra, quem tiver a menor quantidade de cartas após o término da última rodada. A metodologia proposta utiliza uma abordagem informacional para a seleção das cartas do baralho. A abordagem informacional à derivabilidade no contexto da LSC consiste em estabelecer os infons – unidades mínimas de informação semântica fraca – de cada sentença, e uma sentença ϕ será derivável de um conjunto de sentenças Γ se, e somente se, cada infon de ϕ também for infon de alguma sentença de Γ; portanto, a abordagem informacional explicita o caráter não-ampliativo das derivações da LSC. A metodologia proposta conforma-se a critérios de alcançabilidade de cada carta e de balanço entre elas. O critério de alcançabilidade consiste em garantir, para cada carta do baralho, a existência de uma terna de outras cartas da qual ela seja derivável; na metodologia proposta garante-se uma condição ainda mais restritiva, a saber, para cada carta do baralho há um par de outras cartas da qual ela é derivável. O critério de balanço consiste em utilizar os cinco conetivos usuais – negação, conjunção, disjunção, implicação material, equivalência material – de tal modo que a frequência destes conetivos no conjunto das cartas do baralho reproduza aproximadamente a frequência deles nas sentenças em manuais de lógica.

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Revisão da Lógica, Disputas Verbais e Negociações Metalinguísticas

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Resumo

O pluralismo lógico é a visão de que há mais de uma lógica correta. Uma visão particular desta tese é chamada de pluralismo lógico de domínio específico. Aqui se defende que lógica correta ou os conectivos lógicos dependem do domínio de uso, do contexto de uso, ou do pano de fundo linguístico em que o vocabulário lógico é empregado. A dificuldade filosófica, nesta visão, é a de que a comunicação significativa entre lógicos de tradições diferentes e rivais é prejudicada. Parece que toda a comunicação e discussão sobre a revisão de princípios lógicos se transforma em uma mera disputa verbal. Se dois lógicos abordam o mesmo domínio com lógicas diferentes guiando suas investigações, então devem estar usando diferentes conectivos, e, portanto, usando diferentes linguagens, e, consequentemente, falando de coisas diferentes sem perceber isto. Na discussão entre lógicos rivais, podemos pensar que estamos, por exemplo, tendo uma discussão sobre "$\neg A$", mas na verdade estamos usando sentidos diferentes para "$\neg$", de maneira que não falamos de fato sobre a mesma coisa. Se mudamos de linguagem, mudamos de assunto. E não haveria um desacordo de fato. Este problema de comunicação parece impedir desacordos legítimos entre lógicas rivais. Mas como podemos racionalmente justificar nossos princípios lógicos se a própria possibilidade da justificação racional os pressupõem? Como nós podemos fundamentar um conjunto de princípios básicos da razão como o correto sem circularidade ou regresso ao infinito?

Neste trabalho, uma possível solução para este problema é articulada, sem se perder a tese pluralista. Uma solução neopragmatista requer que adotemos uma noção de negociação metalinguística que permita pessoas comunicarem, discordarem e justificarem suas escolhas lógicas mesmo estando em domínios diferentes e usando linguagens distintas. Minha proposta concernente ao problema da justificação e da normatividade na revisão da lógica explora a analogia entre lógica e outras disciplinas normativas, como a ética e o direito.Neste trabalho, apresentamos um método neopragmatista para se pensar a revisão da lógica e a natureza dos desacordos entre lógicos rivais ao enfatizar o caráter antirrealista do vocabulário lógico e o papel normativo que este desempenha em nossas atividades discursivas usuais, especialmente no contexto de negociações metalinguísticas.

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Using the subset construction algorithm to transform imperfect information games into perfect information games

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Abstract

Automata are models of computation that can be used to describe many different situations and has applications in several areas of research [1] For complex tasks, nondeterministic automata appear as a way to describe situations where either multiple paths could or need to be explored to find a successful one. Ben-Ari and Armoni [9], while discussing the teaching of nondeterminism, present an interesting review of many perspectives on the relationship between nondetermismin and concurrency. To these authors, nondeterminism can be discussed in terms of existential semantics, which are suited for automata; and universal semantics, better suited for algorithms. They also quote Lamport [2] and his definitions of nondeterminism and concurrency being two different concepts, although not completely distinct, and his use of two types of temporal logic adds to the argument that they are indeed related, although not the same concept. Regarding concurrency, this is a research that has an important role in computer science, especially due to the advance in distributed systems. Many programming languages with distinct implementations for concurrent programming exist. However, their users are programmers that need to learn these concepts somehow. As Armoni and Ben-Ari show [7, 9], students, while learning concurrency and nondeterminism, might not fully understand these concepts. We showed that programming tools aimed at helping students of various ages to learn programming implement concurrency in different ways [4]. These different perspectives of concurrency, if not well described, could lead to difficulties in program comprehension [7, 9]. To this extent, we propose to discuss this issue from a more formal perspective. Game theory is a theory that provides us with tools to discuss strategic interactions [6]. There are many ways to represent these interactions. Among them, perfect information games allow us to model interactions in which each player chooses a move and he is aware of all previous player's moves. However, there are situations in which players might not be fully aware of other players' choices. These can be modeled as imperfect information games. To represent perfect and imperfect information games, we can use an extensive form representation, in which the game is pictured as a tree. In a previous paper [3], we explored how to represent finite automata (both deterministic and nondeterministic) as games, calling them *A-games*. An A-game has the following formal definition:

Definition 1. (A-game) Let A be a finite automaton $\langle Q_A, \Sigma, \delta_A, q_0, F_A \rangle$ and let G_A be an (extensive) A-game with two players, P_I and P_{II} . In this game, P_I plays states from Q_A and P_{II} plays characters from Σ_A^*.

An A-game runs as follows. P_I , and then P_{II} move, by choosing one state and one symbol to read. According to each state/transition pair, we move in the game tree. The game ends successfully when P_I plays a final state and P_{II} plays the empty character λ. In a previous work [3], we presented proof that, given a game G_A, there are (winning) strategies such that $S(G_A) = L(A)$, where $S(G_A)$ stands for the winning strategies of G_A, and $L(A)$ stands for the language accepted by an automaton A that generated the A-game. This theorem applies both to perfect and imperfect information games.

To create an A-game from a DFA or NFA, we can use the following process:

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```
(1) create a (choice) node and label it P_I.
2. for each state q ∈ Q
    create an outgoing vertex and label it q.
    create a (choice) node and label it P_I.
    attach P_I to vertex q.
    for each w ∈ Q + {λ}:
        create an outgoing vertex and label it w.
        create a (choice) node and label it P_I
        attach P_I to vertex w.
        if σ(q,w) ≠ ∅:
            if |σ(q,w)| = 1:
                q' = σ(q,w)
                create a (choice) node n ∈ P_I. Label it P_I.
                if q' ∈ F:
                    for each w ∈ Q + {λ}:
                        create an outgoing vertex and label it w.
                        if w == λ , mark SUCCESS, else mark FAIL.
                else:
                    go to step 2.
            else:
            if |σ(q,w)| > 1:
                list'_q = σ(q,w)
                create a new information set
                for q' in list_q:
                    create a (choice) node and label it P_I.
                    add it to the information set
                    if q' ∈ F:
                        for each w ∈ Q + {λ}:
                            create an outgoing vertex and label it
                                w.
                            if w == λ , mark SUCCESS, else mark
                               FAIL.
                    else:
                        go to step 2.
        else:
            mark FAIL.
```

In this talk, we would like to further explore A-games by discussing the relationship between nondeterministic and deterministic automata via A-games. Rabin and Scott [5] showed that NFAs could be represented as DFAs, by using the power set construction technique. Considering the concept of A-games and how we can represent both NFAs and DFAs, our research question is: can we represent imperfect information A-games as equivalent to perfect information A-games? Initially, we can observe that according to our method for representing NFAs as A-games, choice nodes that are in a given information set are those from P_{II} , after P_I chose a state. In this situation, P_{II} cannot precisely know which state was chosen, since P_I (which plays states) could have played any state from the information set. Thus, to convert these games from imperfect information to perfect information A-games, we propose to collapse the information sets into one single choice node, by combining actions that enter the information set. This would allow us to remove the "chance" factor which is present in a perfect information game when a player has to decide among choice nodes in an information set [6]. However, when collapsing these choice nodes, we need to also collapse the subtrees from these nodes. Choice nodes that do not end with FAIL will collapse and create new information sets, from which we can reapply the collapsing process, recursively, to eliminate them. A formal definition of conversion from imperfect information A-game to perfect information A-game can also be described:

Definition 2. (Conversion of imperfect information A-game to a perfect information A-game) Let G_I an imperfect information game $\langle N, A, I, \succcurlyeq\rangle$. A perfect information A-game $G_i^p\langle N, A^p, \succcurlyeq\rangle$ from G_I, with A^p is built as follows: for each information set in I, join all the previous action that generates then into one new action.

From our initial observations, although it is possible to generate these game trees, they become quickly bigger than game trees generated from the converted DFA automata, with exponential growth. However, we argue that this shows us the complexity of nondeterminism. In these perfect information A-games, we note that deterministic paths, in which there is no nondeterminism in the original automata, are still present. If we consider the rational aspect of running an automaton, this could characterize that a deterministic path is "rationally" a better choice than a nondeterministic path for these players, even when the nondeterminism (or chance, in the imperfect information game) is eliminated from the game tree. However, it is possible to define a game from the DFA version of a given NFA automaton, in which these deterministic paths do not exist. This happens because when using the subset construction algorithm, we can eliminate unreachable states. Although we can accept the same language from this automata, we removed information on the "rationale" of the automata.

With this work, we want to add to the discussion of game theory and computer science, a topic that has led to many interesting discoveries [8]. Our main goal is to be able to further explore the concept of nondeterminism and its related topics, such as concurrency and parallelism, by providing formal tools that could allow us to explain these concepts to programmers, both novice and experienced, who might find difficulties in understanding the concept. In this talk, we provided a further explanation of how game theory could be an interesting theory to formalize these concepts and find new insights regarding with, by adding rationality to automata theory.

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Mudança de crenças e Lógicas Hiperintensionais

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Resumo

Arcabouços formais para a Epistemologia precisam satisfazer requirementos dicotômicos: ao mesmo tempo que precisam ter suficiente estrutura para permitir derivar conclusões sobre fenômenos epistêmicos de interesse, precisam também ser flexíveis o suficiente para codificar diferentes posições concorrentes na literatura filosófica e lógica.

Mudança de Crenças é a área que estuda como um agente racional muda seu estado epistêmico dadas novas informações, possivelmente conflitantes com aquelas atualmente detidas por ele. Um dos paradigmas mais influentes na área foi proposto por Carlos Alchourrón, Peter Gärdenfors e David Makingson em seu trabalho seminal [1] e é comumente referido como teoria AGM. A partir de tal proposta original, muitos trabalhos analisando diferentes operadores de dinâmica de crenças surgiram na literatura para lógicas clássicas e não-clássicas [6,8]. Particularmente, alguns trabalhos recentes [4] mostraram que, para muitas lógicas importantes para a Computação, não existe uma operação de mudança de crença (revisão ou contração) que satisfaça os requerimentos de AGM. Este problema é usualmente atacado propondo operadores de mudança alternativos para lógicas particulares ou adotando operadores menos restritivos, que são construtíveis em um conjunto maior de lógicas [6].

Desde o trabalho de Hintikka [7], a tradição em Lógica Epistêmica tem favorecido uma modelagem de agentes fortemente idealizados. Uma das críticas a essa modelagem recai sobre a natureza intencional das crenças de um agente em tal modelo, c. f. o trabalho de Cresswell [3], o que leva ao agente acreditar em tudo que pode aquilo que pode ser logicamente dedutível de suas crenças. Tal propriedade não é, entretanto, razoável quando descrevemos agentes epistêmicos reais.

Em Mudança de Crenças, idealizações similares estão presentes nas representações do estado epistêmico de um agente. AGM, por exemplo, representam o estado epistêmico do agente por um conjunto fechado por consequência e, portanto, o agente é logicamente onisciente. Hansson [6] critica o uso de conjuntos fechados no teoria AGM e propõe que uma representação fidedigna deve levar em consideração as crenças explícitas do mesmo. Assim como AGM, entretanto, os operadores de Hansson satisfazem o postulado da extensionalidade, que diz que se duas fórmulas φ e ψ, são logicamente equivalentes, então a resultado de mudar o estado de crenças do agente com a informação φ deve ser igual ao de mudar com a informação ψ. Em outras palavras, Hansson, assim como AGM, valida que mudanças de crença são operações intensionais, e não hiperintensionais.

Recentemente, trabalhos como o de Berto [2] e Souza [9] debruçaram-se sobre operações de mudança de crença que possuem comportamentos hiperintensionais, i.e. são capazes de modelar diferenças hiperintensionais entre crenças e informações, mantendo ainda certas propriedades desejáveis, do ponto de vista filosófico ou formal. De particular interesse para o nosso trabalho, Souza [9] estuda classes de operações de contração hiperintensionais de crença baseado em ferramentas similares e estabelecendo conexões com trabalhos prévios da teoria AGM. Tal trabalho foi posteriormente expandido por Souza e Wassermann [10, 11], que aplicam tais noções para investigar a definibilidade de operadores de mudança de crença em lógicas não-clássicas.

Nesse trabalho, investigamos operações de contração hiperintensionais de crenças. Para definir a noção de hiperintensionalidade nesse trabalho, utilizaremos uma abordagem baseada na

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Lógica Abstrata. O motivo de tal escolha reside, primeiramente, no fato de que assim nossos métodos se aproximarem mais fidedignamente do arcabouço utilizado na teoria AGM, permitindo a comparação de nossos resultados com os da literatura. Adicionalmente, isso nos permite uma "neutralidade" quanto a natureza da hiperintensionalidade trabalhada aqui, que nos permite conectar nosso arcabouço a diferentes posições na literatura. Vamos então introduzir a noção de lógica hiperintensional que utilizamos nesse trabalho.

Definição 1. *Chamamos de lógica hiperintensional consistente uma tupla* $\mathcal{L} = \langle L, Cn \rangle$*, com* L *uma linguagem lógica e* $Cn, C : 2^L \to 2^L$ *operadores de consquência lógica, tais que para cada* $\Gamma \subseteq 2^L$*,* $C(\Gamma) \subseteq Cn(\Gamma)$*. Chamamos* Cn *o operador intensional de* $\mathcal{L}$ *e* C *seu operador hiperintensional.*

A seguir, nos referiremos a lógicas hiperintensionais consistentes simplesmente como lógicas, já que nos limitamos a análise de tais lógicas nesse trabalho. Dizemos que uma lógica satisfaz uma dada propriedade se seu operador intencional a satisfaz, e.g. $\mathcal{L}$ é compacta se Cn o for, e que satisfaz tal propriedade hiperintensionalmente se seu operador hiperintensional a satisfaz, e.g. $\mathcal{L}$ é hiperintensionalmente compacta se C é compacta.

Para construir operadores de mudança de crença, AGM definem a noção de conjunto de restos (em inglês, *remainder set*), que descreve as formas que uma informação pode ser removida de um conjunto de crenças, aqui representadas como fórmulas da linguagem-objeto, de forma significativa.

Definição 2. *Seja* $B \subseteq L$ *um conjunto de fórmulas e* $\varphi \in L$ *uma fórmula da lógica, o conjunto de restos de* B *ao remover* φ *é o conjunto* $B \perp \varphi = \{B' \subseteq B | \varphi \notin Cn(B') \text{ e } B' \text{ é maximal}\}$

Sabemos que para toda lógica monótona e compacta, um conjunto consistente e não-vazio B e uma fórmula não-tautológica φ quaisquer admitem restos, i.e. o conjunto $B \perp \varphi$ é não-vazio. O mesmo não vale, em geral, para lógicas não-compactas ou não monótonas. Assim, operadores de contração por intersecção parcial, que dependem de tais conjuntos de resto, são definíveis em geral apenas para lógicas monótonas e compactas [5]. Vamos estender a noção de conjunto de restos para que possamos contruir operadores de contração para lógicas não-compactas.

Definição 3. *Seja* $\mathcal{L}$ *uma lógica monótona e compacta,* $B \subseteq L$ *um conjunto de fórmulas e* $\varphi \in L$ *uma fórmula* $\mathcal{L}$*. Nós definimos o conjunto de restos hiperintensionais de* B *por* φ *com o conjunto:* $B \perp^C \varphi = \{B' \subseteq B \mid \varphi \notin C(B') \text{ e } \exists B'' \in B \perp \varphi \text{ t.q. } B'' \subseteq B'\}$

Podemos então definir a noção de contrações hiperintensionais de base, como de costume.

Definição 4. *Seja* $\mathcal{L}$ *uma lógica monótona e compacta e* $B \subseteq L$ *um conjunto de fórmulas. Chamamos de operação hiperintensional de mudança de base por intersecção parcial no conjunto* B *uma função* $- : 2^L \times L \to 2^L$ *para qual existe uma função de seleção*[1] γ *em* B*, tal que para qualquer* $\varphi \in L$*:* $B - \varphi = \bigcap \gamma(B \perp^C_{\mathcal{L}} \varphi)$*.*

Para caracterizar tal operação, empregaremos os seguintes postulados.

(inclusão) $B - \varphi \subseteq B$

(C-sucesso) Se $\varphi \notin C(\emptyset)$, então $\varphi \notin C(B - \varphi)$

(uniformidade hiperintensional) Se para quaisquer $B', B'' \subseteq B$, vale que

1. $\varphi \in Cn(B')$ sse $\psi \in Cn(B')$
2. $\varphi \notin Cn(B')$ e $\varphi \in C(B' \cup B'')$ implica que existe algum $B''' \subseteq B$ t.q. $B'' \subseteq B'''$, $\psi \notin Cn(B''')$ e $\psi \in C(B''' \cup B'')$
3. $\psi \notin Cn(B')$ e $\psi \in C(B' \cup B'')$ implica que existe algum $B''' \subseteq B$ t.q. $B'' \subseteq B'''$, $\varphi \notin Cn(B''')$ e $\varphi \in C(B''' \cup B'')$

então $B - \varphi = B - \psi$

(relevância hiperintensional) Se $\psi \in B \setminus B - \varphi$, existe algum $B' \subseteq B$ t.q. $B - \varphi \subseteq B'$, $\varphi \notin C(B')$ mas $\psi \notin B'$ e $\varphi \in Cn(B' \cup \{\psi\})$, e existe $B'' \subseteq B'$ t.q. $\varphi \notin Cn(B'')$ mas $\varphi \in Cn(B'' \cup \{\xi\})$ para qualquer $\xi \in B \setminus B''$.

Com tais postulados, podemos caracterizar as operações de contração hiperintensional de base.

[1] Dizemos que $\gamma : 2^{2^L} \to 2^{2^L}$ é função de seleção em B se (i) $\emptyset \subset \gamma(X) \subseteq X$ e $\gamma(\emptyset) = B$.

Teorema 5. *Sejam $\mathcal{L}$ uma lógica monótona, compacta e hiperintensionalmente monótona e $B \subseteq L$ um conjunto de fórmulas. O operador $-$ é contração hiperintensional de base por intersecção parcial em B sse $-$ satisfaz (inclusão), (C-sucesso), (uniformidade hiperintensional) e (relevância hiperintensional).*

Ademais, podemos ver que tais operações podem ser usadas para definir operações de mudança de crença em lógicas não-clássicas e outras lógicas de interesse para a Ciência da Computação e Inteligência Artificial.

Lema 6. *Sejam $\mathcal{L}_h = \langle L_h, C_h \rangle$ a Lógica Proposicional de Horn e $B \subseteq L_h$ um conjunto de fórmulas de Horn. Uma operação $- : 2^{L_h} \times L_h \to 2^{L_h}$ é uma contração de base de Horn em B sse existe uma contração hiperintensional sobre lógica proposicional clássica $\mathcal{L}_0$ no conjunto B, $\ominus : 2^{L_0} \times L_0 \to 2^{L_0}$, t.q. para qualquer $\varphi \in L_h$: $B - \varphi = B \ominus \varphi$*

Podemos também usar tal construção para estudar operações de mudança de crença em lógicas não-clássicas que, até onde saibamos, não possuiam caracterização completa até o momento [8].

Lema 7. *Sejam $K \subseteq L$ um conjunto intuicionisticamente fechado de fórmulas proposicionais, i.e. $K = C(K)$ e $-$ contração hiperintensional de base em K, $-$, sobre lógica proposicional clássica. Se $-$ pode ser construída com uma função de seleção minimamente gulosa, então $-$ uma contração intuicionista em K.*

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Intuitionism, Merleau-Ponty and Embodied Cognitive Science

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Abstract

The intuitionism of Brouwer and Heyting as laid out in [1], [2], [7] and [8] is one of the main currents in the philosophy of mathematics of the 20th century and views mathematics, essentially, as a mental construction and an introspective activity, which is done on an individual level and independent of the language or the symbolism used to express or communicate it or any objective reality. This mental construction is grounded, in a Kantian sense, in our basic intuition of time [1, p. 80] and mathematical truth is a subjective claim made by a mathematician who has realised an appropriate mental construction. For the intuitionist, mathematical truth needs to be "experienced" [2, p. 90].

Merleau-Ponty, a philosopher in the phenomenological tradition, active mainly in the 1940s and the 1950s in France, developed a view of the mind as an embodied entity, inseparable from our living bodies, as opposed to a dualistic view in the tradition of Descartes. Although mathematics was far from his philosophical main interest, his overall philosophical commitments lead him to a view of mathematics. In [12, p. 446-451] he illustrates his view with a description of a proof of a theorem in geometry and in [13, p.119-120] he analyzes an algebraic formula. Discussions of these passages can be found in [6], [5] and [4]. Merleau-Ponty views a mathematical proof "as an act of expression, fundamentally embodied and essentially creative", in the words of Hass, [5, p. 152], (I should note here that "expression" is a rather technical word for Merleau-Ponty and differs from Husserl's use of the word.) - it is not "a deductive process of restating what is already given in the initial [geometric] figure" [5, p.151]. Hass further describes Merleau-Ponty's point of view:

> "[Rather] the conclusion "goes beyond" or "transgresses" that initial figure [...] [which is encountered by the geometer] as a Gestalt - as an open, incomplete, and sense-laden situation - that he or she must try to discover one particular line of meaning that can be pursued toward the conclusion. Merleau-Ponty thus says that reaching the conclusion of a proof, or some breakthrough step along the way, both "transcends and transfigures" the initial situation. It doesn't merely restate what is already given, but rather demands "crystallizing insight" through which some meaning-possibility suddenly "reorganizes" and "synchronizes" what was before a con-*fusion* of meaning, a problem to be solved." [5, p. 151]

Moreover, as Hass further explains, Merleau-Ponty calls geometry a "motor subject", meaning by this that the living body provides stable vectors of organization that inform the process of proof at every step: vectors such as "up", "down", "left", "right", "in", "through", "intersect", and "extend" and algebraic proof equally presupposes the corporeal vectors of temporality such as "next", "succession", and "progression" [5, p.152].

It is surely possible to see parallels between the view of Merleau-Ponty and the intuitionism of Brouwer and Heyting. Both reject the transcendental or platonic existence of mathematical objects and see mathematics as a human construction and activity, rooted in human thought. On the other hand, one difference to be noted is surely that intuitionism sees the mathematical mental construct occurring along "deductive lines of necessity" starting from a basic Kantian intuition of time [5, p. 159], which leads to the theory of intuitionist logic. Merleau-Ponty, on the other hand, views a mathematical proof or the development of a mathematical idea as a creative process of "transgressive insights - insights that, once formalized, retrospectively disguise the open character of the work and the multiple lines of possible development" [5, p. 159],

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that is, which tend to give us *a posteriori* the illusion of a transcendental necessity. Moreover, intuitionism might seem at first glance rather "mentalist" compared to a view of a philosopher who tried to point out the embodied nature of the mind.

In this talk I shall argue that recent empirical evidence from cognitive science sheds a different light on basic tenets of intuitionism and brings the two views of mathematics closer together. Indeed, intuitionism views mathematics as a mental construction on the individual level. Such mental operations are exactly the subject matter of cognitive science, thus I argue that empirical evidence from cognitive science will have a bearing on basic principles of the intuitionist philosophy of mathematics.

Growing evidence of cognitive science shows that mental cognitive processes make use of neural circuits of the sensory-motor system [3], [14]. This evidence comes from the conceptual metaphor theory of cognitive linguistics as laid out in [9] (and more specifically relating to mathematics in [10]), which shows that many abstract concepts are conceptualized metaphorically via concrete concepts that can be perceived or enacted via the sensory-motor system. Additional evidence comes from cognitive psychology [11] and the theory of "fictive motion" [15], that show that the process of cognition of action verbs, such as "construct" and "experience" - used in intuitionist texts in expressions such as "You describe your activity as mental construction, Mr. Int" [8, p. 75] or "these truths have been experienced" [2, p. 92] - activate sensory-motor areas of our brain. I should draw attention to the fact that such action verbs are ubiquitous in mathematics in expressions such as "the sequence is increasing" or "the function is approaching a limit". Such expressions make use of motion verbs, although no actual motion is occurring, as sequences and functions are obviously static entities.

I would like to point to ways in which this evidence has significance for the basic tenets of intuitionism. In [1, p. 80], Brouwer states that after having abandoned Kant's apriority of space, intuitionism started

> ...adhering the more resolutely to the apriority of time. This neo-intuitionism considers the falling apart of moments of life into qualitatively different parts, to be reunited only while remaining separated by time, as the fundamental phenomenon of the human intellect...

Thus Brouwer bases his philosophy on a basic intuition of time in a Kantian sense. According to recent evidence from cognitive linguistics, in particular conceptual metaphor theory, we conceptualize time through spacial metaphors [9]. For example, "Christmas is coming" or "We are coming up on Christmas" are common place in our language; in the first example we conceptualize ourselves as remaining motionless and Christmas as moving towards us, in the second example we are conceptualizing ourselves moving towards a fixed "place", called or "occupied" by Christmas. Interestingly, immediately after Brouwer states that intuitionism is based fundamentally on an apriority of time, he talks about about time via a spacial metaphor: "the falling apart of moments of life". Therefore, I would argue that Brouwer's intuitionism is actually based on fundamental spacial (rather than temporal) concepts, not in a Kantian sense, but rather in a sense of an "embodied spaciality", i.e. spacial concepts relative to our own bodies in the sense of Merleau-Ponty, illustrated above in claims that geometry, for example, is a "motor subject".

Thus I argue that this evidence pointing to the embodied nature of the mind sheds a different light on intuitionism, turning it, at least partly, into an "embodied philosophy of mathematics" - not because of changing tenets of intuitionism, but simply because of empirical evidence about what the mind is and what a mental construction is. Moreover, an embodied nature of the mind would not go against the basic principle of intuitionism that view mathematics as a "mental construction" - at most it would redefine what exactly "mental" means, i.e. it would redefine the mind as an "embodied mind". On the contrary, in [7, p. 72] writes that "a mathematical theorem expresses a purely empirical fact, namely the success of a certain construction" - how could such a view of mathematics be incompatible with cognitive operations using sensory-motor circuits? Moreover, the mind as embodied brings the intuitionism of Brouwer and Heyting closer to Merlau-Ponty's philosophy.

Keywords. Intuitionism, Brouwer, Heyting, Merleau-Ponty, Conceptual Metaphor, Fictive Motion, Embodied Cognition.

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Teorema do ponto-fixo no contexto da computação e da lógica

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Resumo

Temos como objetivo analisar comparativamente o teorema do ponto-fixo através de um trajeto: 1) lambda calculus não-tipado, 2) funções recursivas e 3) sistemas formais. Faremos uma análise simultânea das provas, comparando nomenclatura e conceitos. Após mostraremos algumas consequências tanto no âmbito da computação quanto no da lógica. O seguinte trabalho não possui resultados novos, mas pretende revisar de forma didática a literatura.

Teoremas de ponto-fixo aparecem em diversas áreas da matemática: análise, topologia, geometria, etc... No âmbito da computação teórica e da lógica, este resultado também ocorre. E ocorre de maneira paradigmática, pois é responsável por garantir o conteúdo autorreferente presente em teoremas clássicos como: teorema da diagonal de Cantor, paradoxo de Russell e teorema da incompletude de Gödel[1].

Weber, em seu livro Computability Theory [2], ao introduzir uma das versões da prova deste teorema se refere a ela como a prova "mágica". A alusão é ao pequeno tamanho que normalmente a demonstração toma e a dificuldade de se ter uma intuição. Por outro lado, a situação é a contrária para alguém que está familiarizado com a prova no contexto do lambda calculus não tipado, a apresentação se torna muito mais clara.

A diferença de dificuldade entre as provas se dá porque há muitos problemas conceituais que devem ser superados para chegar ao teorema do ponto-fixo tanto na sua versão para funções recursivas, quanto na sua versão para sistemas formais. Um dos problemas é a diferença de nível entre números e funções que ocorre tanto em funções recursivas quanto na lógica. Por outro lado, para o lambda calculus não tipado, todos os termos podem ser interpretados como funções puras cujas operações obedecem regras simples, o que ajuda a desmistificar a situação.

Em teoria dos conjuntos, um ponto-fixo para uma função é simplesmente a existência de um ponto $c \in X$ em que a função $f : X \to X$ é tal que $f(c) = c$. Nem sempre uma função $f : X \to X$, com $|X| \geq 2$, possui ponto-fixo, basta pensar em uma função que faz o seguinte, para $x, y \in X$:

$$f(x) = \begin{cases} y, \text{ se } x \neq y \\ \text{qualquer outro elemento de } X \text{ , se } x = y. \end{cases}$$

Em especial, se $X = 2 = \{0, 1\}$ a única função que não possui ponto-fixo será a função negação: $\neg(0) = 1$ e $\neg(1) = 0$.

Porém a situação é diferente em outros contextos que possuem mais estrutura, como no contexto das funções computáveis ou no sistema formal da aritmética. Sempre é possível afirmar a existência de ponto-fixos para certos conjuntos de funções ou fórmulas. O teorema do ponto-fixo postula a existência de um objeto que intuitivamente diz[2]:

1. **lambda calculus:** "eu (programa $\mathcal{Y}f$) me autotransformo de acordo com a função f"
2. **funções recursivas:** "eu (programa $\{n\}$) me autotransformo de acordo com a função f"
3. **aritmética:** "eu (sentença A) sou equivalente a fórmula F aplicada a mim mesma"

Com a construção deste objeto conseguimos provar a existência de um ponto-fixo. Os enunciados em cada contexto são:

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[1] Uma apresentação destes resultados à luz do ponto-fixo em um ponto de vista generalizado pode ser conferido em [4].

[2] Requisitos sobre como definir tais sentenças podem ser conferidos em [1]. Temos como referência para o sistema aritmética os axiomas para a aritmética de Robinson, ou qualquer extensão desta.

1. **lambda calculus:** Para toda função f , existe um ponto-fixo $\mathcal{Y}f$ tal que: $\mathcal{Y}f =^{\beta} f(\mathcal{Y}f)$.
2. **funções recursivas:** Para toda função f, existe um ponto-fixo n tal que: $\phi_n \simeq \phi_{f(n)}$.
3. **aritmética:** Se T é uma teoria que pode representar funções e predicados recursivos, então para toda fórmula $F(x)$, existe uma fórmula A tal que: $\vdash_T A \leftrightarrow F(\ulcorner A \urcorner)$.

Tomando [1] como referência, podemos verificar que as provas destes teoremas são simétricas. Uma vez estabelecido resultados preliminares e tomado cuidado para separar os níveis em questão envolvidos na prova das funções recursivas e da aritmética, podemos resumir a demonstração na seguinte tabela:

	lambda calculus	funções recursivas	aritmética
1	$\Delta := \lambda x.xx$	$\delta(x) :\simeq \phi_x(x)$	$D(\ulcorner X \urcorner) \leftrightarrow: X(\ulcorner X \urcorner)$
2	$\Delta_f := \lambda x.fxx$	$\phi_{\phi_d(x)} :\simeq \phi_{f\circ\delta(x)}$	$D_F(\ulcorner X \urcorner) \leftrightarrow: F(\ulcorner D(\ulcorner X \urcorner) \urcorner)$
3	$\Delta_f\Delta_f = \lambda x.f(xx)\Delta_f =^{\beta} f\Delta_f\Delta_f$	$\phi_{\phi_d(d)} \simeq \phi_{f\circ\delta(d)} \simeq \phi_{f(\phi_d(d))}$	$D_F(\ulcorner D_F \urcorner) \leftrightarrow F(D(\ulcorner D_F \urcorner)) \leftrightarrow F(D_F(\ulcorner D_F \urcorner))$
4	$\mathcal{Y}f := \Delta_f\Delta_f$	$n := \phi_d(d)$	$A \leftrightarrow: D_F(\ulcorner D_F \urcorner)$
$\therefore$	$\mathcal{Y}f =^{\beta} f(\mathcal{Y}f)$	$\phi_n \simeq \phi_{f(n)}$	$A \leftrightarrow F(\ulcorner A \urcorner)$

O passo em que o maquinário da prova está realmente operando é o de número 3. Porém, temos que dar alguma justificativa de porque as definições do passo 1 e 2 não vão gerar problemas para nós. Em resumo, nos passos 1 e 2, nas funções recursivas, δ não pode ser total se for pensada fora dos índices; e também na aritmética D não pode ser definida sem tornar inconsistente o sistema formal se deixada fora dos números de Gödel. A razão disto é que no caso dos sistemas formais temos como consequência a seguinte contradição:

$$D(\ulcorner \neg D \urcorner) \leftrightarrow \neg D(\ulcorner \neg D)$$

Já no contexto das funções recursivas temos a seguinte contradição, assumindo que o $f \circ \delta$ tem número e em uma enumeração de funções recursivas:

$$f(\delta(e)) = \phi_e(e) = \delta(e)$$

que é uma contradição, pois como vimos, nem toda função possui ponto-fixo.

Por outro lado, como o lambda calculus não tipado não faz esta diferença de níveis[3], não há este problema conceitual e a prova se torna direta.

É possível ver que tomando F como o predicado de não provabilidade $\neg P$, $A \leftrightarrow \neg P(\ulcorner A \urcorner)$ é a formulação usual da sentença de Gödel, que diz de si mesma que não é provável, implicando assim o teorema da incompletude. Outras consequências podem ser tiradas deste teorema: teorema de Rice e outros teoremas limitantes. De modo geral, tais argumentos podem ser vistos como instanciação do teorema realizado em teoria das categorias provado por Lawvere em [3] e explorado por [4].

Palavras-chave. ponto-fixo, computabilidade, sistema-formal.

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Countable powers of countably pracompact groups

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Abstract

Throughout this abstract, every topological space will be Tychonoff (Hausdorff and completely regular) and every topological group will be Hausdorff (thus, also Tychonoff). Recall that an infinite topological space X is said to be

- *pseudocompact* if each continuous real-valued function on X is bounded;
- *countably compact* if every infinite subset of X has an accumulation point in X;
- *countably pracompact* if there exists a dense subset D in X such that every infinite subset of D has an accumulation point in X.

We also recall below the definition of *selective ultrafilter* on ω:

Definition: A selective ultrafilter on ω is a free ultrafilter p on ω such that for every partition $\{A_n : n \in \omega\}$ of ω, either there exists $n \in \omega$ such that $A_n \in p$ or there exists $B \in p$ such that $|B \cap A_n| = 1$ for every $n \in \omega$.

The existence of selective ultrafilters is independent of ZFC. In fact, Martin's Axiom (MA) implies the existence of $2^{\mathfrak{c}}$ selective ultrafilters [10], while there is a model of ZFC in which there are no selective ultrafilters [11].

Pseudocompactness is not preserved under products for arbitrary topological spaces [9], but interestingly Comfort and Ross proved that the product of any family of pseudocompact topological groups is pseudocompact [8]. This result motivated Comfort to question whether the product of countably compact groups is also countably compact. More generally, he asked the following question in the survey book *Open Problems in Topology*:

Question 1: Is there, for every (not necessarily infinite) cardinal number $\alpha \leq 2^{\mathfrak{c}}$, a topological group G such that G^γ is countably compact for all cardinals $\gamma < \alpha$, but G^α is not countably compact?

Douwen was the first to prove consistently (under MA) that there exists two countably compact groups whose product is not countably compact [7]. In 2005, **Question 1** was answered positively in [6], assuming the existence of $2^{\mathfrak{c}}$ selective ultrafilters and that $2^{\mathfrak{c}} = 2^{<2^{\mathfrak{c}}}$. Finally, in 2021, it was proved in ZFC that there exists two coutably compact groups whose product is not countably compact [5].

It is natural also to ask productivity questions for countably pracompact groups. In this regard, Bardyla, Ravsky and Zdomskyy constructed, under MA, a Boolean countably compact topological group whose square is not countably pracompact [4]. More generally, one can ask Comfort-like questions for countably pracompact groups:

Question 2: For which cardinals $\alpha \leq 2^{\mathfrak{c}}$, there exists a topological group G such that G^γ is countably pracompact for all cardinals $\gamma < \alpha$, but G^α is not countably pracompact?

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In [3], under the assumption of CH, the authors showed that for every positive integer $k > 2$, there exists a topological group G for which G^k is countably compact but G^{k+1} is not countably pracompact (in fact, in their paper, such G^{k+1} was not even selectively pseudocompact).

In this talk, we present an answer to **Question 2** in the case $\alpha = \omega$, assuming the existence of $\mathfrak{c}$ selective ultrafilters.

Keywords. Topological group, countable pracompactness, countable compactness, selective pseudocompactness.

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Posters
Pôsteres

Lógicas Modais para Atitudes de Engajamento

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Resumo

Investigamos a possibilidade de modelar processos que incluem a dinâmica entre comunicação, conhecimento e enviesamento de um agente a partir de uma perspectiva lógico-formal. Para isto, é explorado o potencial para construir uma lógica modal capaz de capturar noções relacionadas a posicionamentos pessoais, tendo como eixo central a ideia de engajamento de um agente com respeito a uma informação apresentada. Como modalidade primitiva, propomos formalizar a noção de 'aprovação', caracterizando esta atitude como determinada pelo relacionamento entre o universo de inclinações pessoais e as crenças de um agente e, a partir desta, são definidas formalmente modalidades correlacionadas, como 'tolerância', 'polarização', dentre outras. Uma das principais motivações para tal tarefa é investir na possibilidade de abstrair ou isolar, na forma de um operador modal, o componente não-cognitivo de expressões que envolvem atitudes extra-epistêmicas, as quais são normalmente confundidas com atitudes estritamente epistêmicas. Pretendemos assim, evitar as ambiguidades e dificuldades comuns na caracterização de sentenças que expressam também atitudes não-cognitivas, consequentemente permitindo a formalização e análise lógica das mesmas. Estes componentes não-cognitivos são principalmente atitudes motivacionais, aquelas cuja direção de ajuste, na terminologia de Searle (1979), assumiriam um movimento do mundo para a mente. Uma relação entre agentes e proposições que também envolva aspectos motivacionais demanda como componentes semânticos elementos contextuais, os quais influenciam tanto a observação do agente como a atribuição de sua atitude proposicional. O aspecto formal desta modelagem inclui utilizar semânticas topológicas, estruturas que permitem formalizar as intuições mencionadas e também a ideia de proximidade, seguindo a inspiração humeana de que agentes podem se posicionar a partir do que conseguem observar em suas vizinhanças epistêmicas, influenciados por relação de contiguidade e similaridade. Estas vizinhanças, quando incluídas em um modelo constituído também por um conjunto de disposições prévias, auxiliam a formar um cenário que chamamos aqui, inspirados em Rozeboom (1967), Woods (2005) e Pariser (2011), de "bolha de engajamento". Em nossa proposta, as lógicas para engajamento inicialmente são concebidas como extensões das SSL (Subset Space Logics) (ver Moss & Parikh, 1992; Ditmarsch et al., 2015, 2017), lógicas que interpretam epistemicamente estruturas de espaços subconjuntos. Introduzimos então uma extensão doxástica, baseada nos sistemas epistêmico-doxásticos topológicos desenvolvidos em Bjorndahl & Özgün (2019), e, por fim, introduzimos o operador modal de aprovação e seu respectivo aparato semântico, o qual inclui a adição de um conjunto de vizinhanças - chamadas desideratas - ao espaço topológico que interpreta a lógica epistêmico-doxástica de base. Cada fórmula da linguagem da lógica de engajamento é então interpretada levando em conta um cenário epistêmico-doxástico composto pelo estado atual, um conjunto de observações (ou alcance epistêmico) e um alcance doxástico, acrescidos de um conjunto de desideratas (mundos possíveis desejados). As aplicações de sistemas lógicos construídos para tal finalidade incluem a análise de fenômenos contemporâneos como polarizações políticas e ideológicas alimentadas por notícias falsas, a dinâmica subjacente ao comportamento engajado de agentes, ao entrincheiramento e às bolhas epistêmicas, análise das câmaras de eco e o fenômeno do Big Data.

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Equivalências em jogos topológicos seletivos em classes de subconjuntos densos sobre o espaço das funções contínuas $C_k(X)$

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Resumo

Jogos topológicos seletivos foram formalizados em 1974 no trabalho [7]. Ao longo dos anos, muitas variações dos jogos topológicos seletivos clássicos foram aparecendo, junto com diversas relações com princípios seletivos.

Fazendo generalizações de uma variedade de resultados, principalmente do resultado principal de [4], obtemos equivalências das distintas variações dos jogos topológicos seletivos quando consideramos a classe dos subconjuntos densos no espaço das funções contínuas munido da topologia compacto-aberto.

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A pedagogical experiment with semantic-oriented refutation systems for a two-sorted first-order logic of graphs

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Abstract

Interactive theorem provers are computational systems that assist humans with the development of formal proofs. Using a proof assistant as a didactic tool can not only guarantee the correctness of proofs written by students before they are assessed and give them immediate feedback, but also improve pen-and-paper solutions [1]. In this context, ProofWeb [2] allows its users to interact with Coq[1] through an IDE (Integrated Development Enviroment) to practice natural deduction for both propositional and first-order logics. Although a refutation system via the semantics for classical propositional logic has already been designed [3, 4], no implementation of a similar system has yet been provided for the first-order case. We propose thus a refutation system via the semantics for classical first-order logic, to be used by students attending a 30 hours undergraduate introductory course on first-order logic. This pen-and-paper system is intended as a prototype for later computerized implementation in Coq. The target audience has a strong background in Discrete Math and has already used the above-mentioned ProofWeb tools, integrated within the Moodle course management system.

Using the proposed system, the participants are supposed to verify the refutability of conjectures about graphs. Specifically, we envisage that their tasks consists of four phases:

1. translation of the properties about graphs described in natural language in a given individualized (refutable) conjecture into appropriate formulas of a two-sorted first-order language;
2. verification 'by hand' of the equivalence between the formulas previously described and the ones we present as the references for each property they are working on;
3. construction of an adequate countermodel for the given conjecture;
4. formal verification of the satisfaction (or not) of the reference formulas by this model (assuming that it works) via semantical refutation trees.

After the currently undergoing experiment is concluded, our main interest will be to understand, from the viewpoint of the students, the benefits of having a method that helps formulating more structured verifications of the refutability of first-order conjectures. As future work, we aim to implement a ProofWeb module for the proposed system and to formalize a heuristics for building countermodels for refutable first-order logic conjectures. Such countermodels will play a role in the second phase of the task described above.

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Árvores e Propriedade D

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Resumo

O estudo foca em compreender árvores por meio da topologia de intervalos, que é definida pela sub-base $\{\uparrow t : t \in T\} \cup \{\downarrow t : t \in T\}$ [7], ao longo do texto topologia na árvore se referirá a essa. A questão central é a busca por uma caracterização por meio da própria ordem para quando uma árvore tem a propriedade topológica D, definida em [9], e possíveis implicações.

Em [6] foi mostrado que a árvore de fechados de um estacionário co-estacionário de ω_1 é um espaço onde todo subespaço não enumerável tem fechado discreto de mesma cardinalidade, mas não é D, respondendo à Questão 3.6 de [2]. Temos que toda árvore fechada por caminhos é D, assim como toda árvore especial. Foi provado em [4] que para uma classe de ordens lineares $\mathbb{L}$ tal que toda árvore $\mathbb{L}$-imersível (ou seja, existe função da árvore até a reta que preserva ordem) é hereditariamente D. O artigo [5] expandiu essa classe, mas também mostrou que as árvores imersíveis nelas tem ramos enumeráveis, assim não contemplamos todas as árvores possivelmente D, visto que todo ordinal é uma árvore e [10] mostra que basta cofinalidade enumerável para ter a propriedade D.

Provamos que uma árvore é D se, e somente se, ela é aD (definição em [1]), logo toda árvore paracompacta é D, verificamos também que existe árvore D não meta-Lindelöf. Na falta de caracterizações para D estudamos para propriedades relacionadas, e caracterizamos a propriedade linearmente D (e linearmente Lindelöf) para árvores por meio da cofinalidade dos seus ramos e possíveis sub-árvores.

Muitas perguntas ainda restam a respeito de árvores com a propriedade D, se podem ser caracterizadas por imersões em ordens lineares, e se as caracterizações de linearmente D e linearmente Lindelöf podem ser adaptadas para encontrar outras propriedades desses espaços.

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Espaços de Michael e Pequenos Cardinais

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Resumo

Pequenos cardinais são cardinais que se encontram entre ω_1 e $\mathfrak{c}$. No geral, são definidos como o menor tamanho de uma família de ω^ω, $[\omega]^\omega$ ou de subconjunto dos reais, em que deixamos de garantir alguma propriedade. De particular interesse para nós, serão o menor cardinal tal existe uma família ilimitada em ω^ω, $\mathfrak{b}$; menor tamanho de uma família cofinal em ω^ω e o menor numero de subconjuntos magros dos reais que precisamos para cobrir o espaço todo. Obviamente, CH nos dá que todos estes são iguais à ω_1. No entanto, é consistente que eles sejam diferentes cardinais. Tomando como hipótese certas igualdades, podemos construir certos espaços topológicos.

Nos anos 70, Ernest Michael publicou o artigo [5] dando vários contra-exemplos a respeito da preservação da normalidade em espaços produto. Em particular, usando CH, ele construiu um espaço de Lindelof tal que o produto com os irracionais não é de Lindelof. Até hoje, não sabemos se tal espaço existe em ZFC, porém a condição de CH foi consideravelmente enfraquecida para igualdades entre pequenos cardinais. O espaço dos irracionais, além de subconjunto dos reais, é equivalente ao espaço ω^ω. Com esta caracterização, fica mais fácil de ver a relação que o problema de Michael pode ter com pequenos cardinais.

Lawrance em [4] expandiu a construção original de Michael e mostrou que a técnica usada está limitada a modelos em que vale $\mathfrak{b} = \omega_1$. Isto poderia apontar que o problema é independente de ZFC. No entanto, no mesmo ano, [2] teve Alster mostrando que outros tipos de construções são possíveis. Especificamente, ele construiu um espaço de Michael usando $\mathfrak{b} = \mathfrak{d} = cov(\mathcal{M})$, ou seja, dentro da possibilidade de termos $\mathfrak{b} > \omega_1$.

Já no final da década, em 99 no artigo [6], Justin Moore enfraqueceu as condições que garantem a existência de um espaço de Michael. Mais especificamente, ele introduziu o conceito de sequência de Michael, que ele usa na construção, e utilizar $\mathfrak{d} = cov(\mathcal{M})$ para construir tal sequência. Moore também notou que todos estes espaços vivem em um produto de um ordinal com o espaço de Cantor.

Nosso objetivo será explorar principalmente estas construções, pois se tratam das principais. Atualmente, começaram a notar conexões que estes espaços tem com uma classe chamada de espaços produtivelmente Lindelof e com jogos topológicos. Tais conexões foram usadas, por exemplo, em [1] para unificar as abordagens de Lawrance e Alster.

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Extraction of infons in finite many-valued logics

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Abstract

The logical work can alternatively and, from a didactic point of view, advantageously be performed with infons (atoms of weak semantic information) (see [5]). The implementation of an informational approach to logics offers, in comparison to the current use of the notion of truth, at least two important benefits. First, this approach supports both the refinement of valid arguments (by removing spurious information) and the correction of deductively invalid arguments (by strengthening the premises through the inclusion of information, or weakening the conclusion through the exclusion of information). Second, as will be shown, the inspection of the validity of an argument will depend, in general terms, on the presence of minimum units of information (infons) in the conclusion and in the premises. This relation between the presence and the absence of infons in the premises and in the conclusion of an argument helps to make explicit the non-amplifying character of the classical deductive validity.

There are at least two notions of weak semantic information: a positive one - e.g., the conjunctives (clauses) of the conjunctive (clausal) normal form in the Classical Sentential Logic -, and a negative one - e.g. the disjunctives of the disjunctive normal form (see [1] and [2]). We propose a procedure for the extraction of negative infons from Finite Many-Valued Logics. Let L be a Fiinite Many-Valued Logic, V the set of its distinguished values, and ϕ a formula of L. The procedure for the extraction of infons consists of two steps. The first step consists of obtaining the rules for the labelled analytic tableaux for L. We use a technique proposed by Carnielli (see [3]), which consists of applying a generalization of Karnaugh Maps to the truth tables of L. In the second step, for each $v \in V$, the complete trees of $v\phi$ are built. The atoms of each open branch are collected into a set. These sets are the infos of ϕ.

There is a version of the well-known Quine-McCluskey method for the efficient minimization of many-valued logical functions (see [4]), but it is useless for the informational approach, since minimum units of information (infons) do not coincide with prime implicants.

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Submodelos elementares, axioma de Martin e árvores geradoras sem ramo infinito*

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Resumo

Submodelos elementares tem aplicações nas mais diversas áreas, incluindo em grafos. Aqui, iremos utilizar submodelos elementares para demonstrar um resultado sobre grafos. Para compreender o enunciado de tal resultado, precisaremos de algumas definições:

Definição 1. *Um grafo $G = (V, A)$ é ω-conexo se é infinito e, dado qualquer s conjunto finito de vértices, $G - s$ é conexo.*

Dado $(P, \leq)$ parcialmente ordenado, temos que:

Definição 2. *Se $p, q \in P$ são tais que $q \leq p$, diremos que q estende p. Se dois elementos tem uma extensão comum, eles são compatíveis - e incompatíveis caso contrário.*

Definição 3. *Um subconjunto de P é uma anticadeia se todos os seus elementos são dois a dois incompatíveis.*

Definição 4. *$(P, \leq)$ satisfaz ccc (countable chain condition) se todas as suas anticadeias são enumeráveis.*

Definição 5. *Um subconjunto D de P é denso se todo $p \in P$ tem extensão em D.*

Definição 6. *Seja $\mathcal{D}$ uma família de densos de P. Um filtro $\mathcal{D}$-genérico é um subconjunto G de P tal que:*

- *se $p \leq q$ e $p \in G$, então $q \in G$*
- *se $p, q \in G$, então eles possuem extensão comum em G*
- *$D \cap G \neq \emptyset$ para todo $D \in \mathcal{D}$.*

Definição 7. *Dado um cardinal κ estritamente menor do que o contínuo, MA_κ, o Axioma de Martin para κ, é a afirmação de que existe um filtro $\mathcal{D}$-genérico para cada conjunto parcialmente ordenado ccc e cada família $\mathcal{D}$ de tamanho κ de subconjuntos densos. Para $\omega < \kappa < \mathfrak{c}$, MA_κ é independente dos axiomas de ZFC.*

Além disso, utilizaremos também conceitos de submodelos elementares:

Definição 8. *Dado G grafo e M submodelo elementar, $G[M]$ é o subgrafo de G induzido por M, ou seja, o conjunto de vértices de G que estão em M e das arestas de G que possuem ambas as extremidades em M.*

Demonstraremos, então, o seguinte resultado:

Proposição 1. *Dado G grafo e M submodelo elementar, se G é ω-conexo, então $G[M]$ também é ω-conexo.*

Com isso, iremos adaptar as demonstrações encontradas no artigo [1] de forma a utilizar ferramentas de submodelos elementares para mostrar o seguinte resultado:

Teorema 1. *Se vale MA_k, então todo grafo ω-conectado de cardinalidade $\leq \kappa$ tem uma árvore geradora se ramos infinitos.*

Palavras-chave. grafos, axioma de Martin, submodelo elementar.

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Referências

[1] Komjáth, P. Martin's Axiom and Spanning Trees of Infinite Graphs. *Journal of Combinatorial Theory* Series B 56:141–144, 1992.